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NASA Contractor Report 3571

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Analysis of Vibration Induced Error in Turbulence Velocity Measurements From an Aircraft Wing Tip Boom

Safwan H. Akkari and Walter Frost

CONTRACT NAS8-34627
JUNE 1982

NASA

NASA Contractor Report 3571

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Prepared for
Marshall Space Flight Center
under Contract NAS8-34627

National Aeronautics
and Space Administration

Scientific and Technical
Information Office

1982

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ACKNOWLEDGMENTS

This research was supported under NASA Contract No. NAS8-34627. The authors are grateful for the support of A. Richard Tobiason of the Office of Aeronautical and Space Technology, NASA Headquarters, Washington, D.C. Special thanks go to Dennis W. Camp of the Space Sciences Laboratory, Atmospheric Sciences Division, NASA/George C. Marshall Space Flight Center, Alabama, who monitored the research program.

ABSTRACT

The effect of rolling motion of a wing on the magnitude of error introduced due to the wing vibration when measuring atmospheric turbulence with a wind probe mounted on the wing tip is investigated. The wing considered has characteristics similar to that of a B-57 Cambera aircraft, and Von Karman's cross spectrum function is used to estimate the cross-correlation of atmospheric turbulence. Although the error calculated is found to be less than that calculated when only elastic bendings and vertical motions of the wing are considered, it is still relatively large in the frequency's range close to the natural frequencies of the wing. Therefore it is concluded that accelerometers mounted on the wing tip are needed to correct for this error, or the atmospheric velocity data must be appropriately filtered.

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LIST OF SYMBOLS

a	distribution of semi-chord
A_{ij}, B_{ij}	aerodynamic cross terms
b	semi-chord (ft)
b_R	reference semi-chord (ft)
b/b_R	distribution of semi-chord
C	Theodorsen's function
E	expected value
EI	beam bending stiffness
F	force on beam
h	impulse response function
H	transfer function
J_o	Bessel function of the first kind order 0
J_1	Bessel function of the first kind order 1
k	reduced frequency
K	Kussner function
ℓ	half-length of the wing (ft)
L	turbulence length scale (ft)
m	mass per unit span along the wing (lbm/ft)
M	bending moment
M_i	generalized mass of the mode
$P(t)$	random process
$p(t)$	ensemble member of a random process $P(t)$
$R(t)$	correlation function
S	area of wing (ft^2)

s	non-dimensional time variable
T_i	amplitude of natural mode
t	time variable (sec)
u	vertical gust velocity (ft/sec)
V	mean flight speed (ft/sec) and shear force on a beam segment
w	vertical displacement of wing
y	spanwise distance (ft)
Y_o	Bessel function of the second kind order 0
Y_1	Bessel function of the second kind order 1
Δ	spanwise element (ft)
η	seperation distance (ft)
μ_i	non-dimensional generalized mass
ξ_m	non-dimensional amplitude of natural mode
ρ	mass density of air (lbm/ft^3)
ϕ_i	ith natural bending mode of the wing
ϕ_p	power spectrum of p(t)
ϕ_σ	power spectrum of output
$\phi_{\sigma p}$	cross spectrum of σ and p
ϕ_m	measured spectrum at wing tip
ψ	slope due to bending
Ω_i	reduced natural frequency
ω	frequency of gust (radians/sec)
ω_i	natural frequency of wing (radians/sec)

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CHAPTER I

INTRODUCTION

The aim of this study is to determine the relative error in turbulent velocity measurements caused by the vibrations of the wing of a B-57 Cambera-type airplane when atmospheric turbulence is measured by a probe mounted on the wing tip. NASA is planning a program to measure turbulence data using a B-57 airplane. Whether or not accelerometers should be mounted on the wing tip in order to obtain accurate measurements is relevant to this program. Reference [1] contains a similar analysis. It uses spectrum analysis to measure the wing tip velocity spectrum and the error introduced in the turbulence measurement due to the wing vibration for the same aircraft. In the aforementioned work, the motion of the airplane was restricted to vertical motion and vertical bending of the wing. The restriction of the airplane to vertical motion and vertical bending while ignoring rolling, may cause an overestimation of the magnitude of the elastic modes. If rolling is taken into account, the magnitudes of the elastic modes will be less due to a relaxation of the elastic modes. In other words, the coupling between the rolling motion and the other types of motion (as will be seen later, the coupling is between the rolling mode and the antisymmetric modes only) may cause the wing to vibrate less than the amount predicted when the airplane is restricted to vertical motion, and hence

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*Numbers in brackets correspond to similarly numbered references in the List of References.

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the relative error in the measured turbulence would be less than that calculated in [1].

A secondary aim of this work is to more fully document the computer program that was used in [1] with the modifications for rolling motion included. The complete computer code with annotations is given in the appendix.

Before tackling the direct problem, a brief discussion of spectrum analysis is given. Since Wiener first published his classical monograph [2] in 1933, spectrum analysis has been increasingly used in wide and diverse scientific areas. That monograph showed for the first time how to use the Fourier integral as a link between two otherwise distinct branches of mathematics--namely, statistics and analysis. The complex form of the Fourier integral theorem states that if f(t) is absolutely integrable on the whole t axis and if f(t) is piecewise smooth on every finite interval, then the following equality holds:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} f(\lambda) e^{i\omega(\lambda - t)} d\lambda \tag{1.1}$$

The function:

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(\lambda) e^{i\omega\lambda} d\lambda \tag{1.2}$$

is called the Fourier transform of f(t). If Equation 1.1 holds for f(t), then we have:

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega) e^{-i\omega t} d\omega \tag{1.3}$$

i.e., f(t) is the (inverse) Fourier transform of $F(\omega)$. The advantage of transforming a function from one domain to another (e.g., from time domain to frequency domain) is that the complicated mathematical operations on the original function (such as convolution and differentiation) are reduced to simple algebraic operations on the transformed function. A brief historical account of Fourier analysis can be found in [3]. Reference [4] is a highly readable book on Fourier analysis.

CHAPTER II

ASSUMPTIONS ABOUT TURBULENCE AND AIRPLANE MOTION

To calculate the response of an airplane to atmospheric turbulence, several elements are needed. These are:

1. The statistical description of the turbulent field (the input.
2. The calculation of the aerodynamic forces associated with the turbulent field (the gust forces).
3. The calculation of the transfer functions which relate the airplane motion (or other quantities of interest) to the gust forces.
4. The combination of the turbulence description with transfer function to obtain the output.

Steps 2 through 4 depend strongly on the method adopted for describing the turbulent field in Step 1. The analysis that follows utilizes the two-point correlation function of the velocity component of interest. The turbulent field is considered to be homogeneous, isotropic, and momentarily frozen (Taylor's hypothesis).

Another important factor in describing the atmospheric turbulence is its randomness in the flight path of the airplane. In the case of what is called one-dimensional gust, the gust field is considered to be random in the direction of the flight but is assumed uniform in the spanwise direction as depicted in Figure 1a. This assumption is satisfactory as long as the ratio of the span of the airplane to the turbulence length scale is small (less than one-tenth). However, if this is large, i.e., the turbulence length scale is small, the gust should also be considered random in the spanwise direction as depicted in Figure 1b.

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(a) One-dimensional gust (b) Two-dimensional gust

Figure 1 Illustration of a one- and two-dimensional gust.

Thus in the case of two-dimensional gusts, the problem of calculating the response of an airplane is essentially one of determining the response of a linear system, i.e., the airplane, to a multi-dimensional stationary random process which in this case is the atmospheric turbulence. The assumptions of linearity and stationarity made in Reference [1] are also made in this study.

The problem of calculating the response of an airplane to gust loads in the general case is a formidable one. To simplify the work required, some assumptions concerning the motion of the airplane are needed. The airplane is restricted to vertical motion and to distortion in the first few free-bending modes of the wing. In addition to vertical motion and vertical bending, the rolling of the wing about its symmetry axis is taken into consideration. The restriction of the airplane to vertical motion and elastic bending of the wing is tantamount to the assumption that the moment of inertia of the fuselage about the x-axis (see Figure 2) is infinite. The inclusion of the rolling motion of the wing and ignoring the rolling motion of the fuselage is

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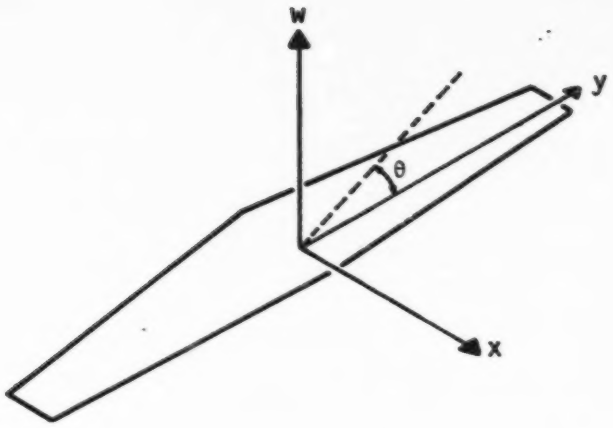

Figure 2 Coordinate system.

equivalent to the assumption that the moment of inertia of the fuselage is negligible. In this case the central part of the wing is taken to have high stiffness so that it can roll but not bend appreciably. The actual wing vibration is somewhere between these two extreme cases, i.e., the actual solution is in between that calculated in Reference [1] and the one calculated in this work. The motions and bendings of the wing considered are those caused by only the vertical component of the gust acting on the wing of the airplane.

According to the preceding simplifications, the pitching and yawing motions of the airplane are ignored. Also, the contribution to roll by the gust acting on the tail is not included in the analysis.

The airplane considered in this work is one that has wing characteristics similar to that of a B-57 wing. The system of axes employed is shown in Figure 2 where the spanwise coordinate, y, is the independent variable and the vertical displacement, w, is the dependent variable. The angle of rotation of the wing about the x-axis is denoted by

θ. The wing is assumed to be a flat plate having stiffness similar to a B-57 wing, and the aerodynamic forces appearing later are those calculated from strip theory. The distributions of the mass and that of the semi-chord across the semi-span are shown in Figures 3 and 4, respectively. For calculating the atmospheric turbulence power spectrum, the von Karman cross spectral function as defined in [5] is assumed.

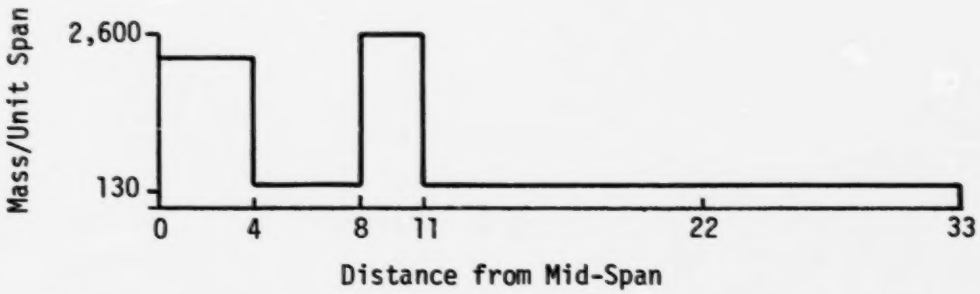

Figure 3 Distribution of mass across the semi-span of the wing.

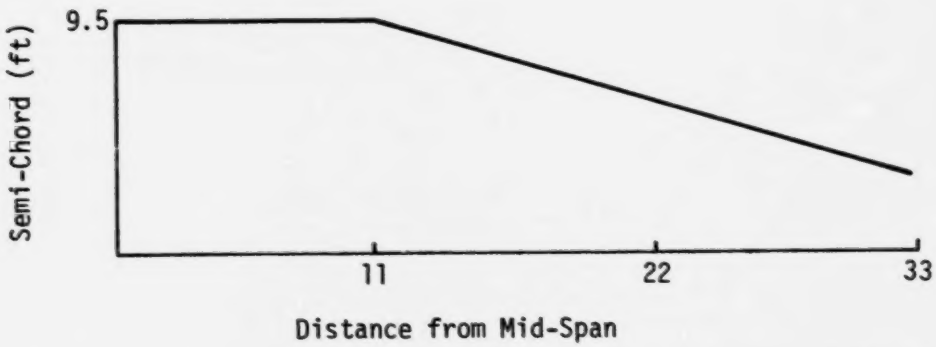

Figure 4 Distribution of semi-chord across the semi-span of the wing.

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CHAPTER III

SPECTRUM ANALYSIS

3.1 Output Spectrum of a Single Input

Consider a random process P(t) and an ensemble member p(t) of this process. The autocorrelation function of P(t) is defined by:

$$R_p(t,t+\tau) = E[P(t)P(t+\tau)] \tag{3.1}$$

where E denotes the expected value. In this case the process P(t) is stationary and the autocorrelation function becomes a function only of time difference τ; that is,

$$R_p(t,t+\tau) = R_p(\tau) \tag{3.2}$$

Note that in practical situations we are usually forced to work with only one ensemble member of a process and, consequently, derive mean value, correlation function, etc. from this member. For this we assume the ergodic theorem, which allows time averages to be equated to the corresponding statistical averages, i.e.,

$$E[P(t)] = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} p(t)dt \tag{3.3}$$

$$R_p(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} p(t)p(t+\tau)dt \tag{3.4}$$

An even more useful statistical characteristic of a random process is its power spectrum which is defined as the transform of its correlation function:

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$$\phi_p(\omega) = \int_{-\infty}^{\infty} R_p(\tau)e^{-i\omega\tau}d\tau \tag{3.5}$$

If p(t) is the input to a linear time invariant system, the response of this system is expressed as a convolution integral of p(t) and an influence function h(t):

$$\sigma(t) = \int_{-\infty}^{\infty} p(\lambda)h(t-\lambda)d\lambda \tag{3.6}$$

For the analysis represented in the next chapter, p(t) represents the gust at one station along the wing and $h(\ell,t)$ is the impulse response at the right wing tip. The Fourier transform of $h(\ell,t)$ will be denoted as $H(\ell,\omega)$ and will be defined as the right wing tip velocity due to a gust located at a station y_1 along the wing. To simplify notation, $h(\ell,t)$ and $H(\ell,\omega)$ will be written as h(t) and $H(\omega)$ with the understanding that they represent the impulse response and the transfer function at the right wing tip. Now, the autocorrelation function of the output can be shown [6] to be related to the correlation function of the input as follows:

$$R_\sigma(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_p(\tau+\lambda_1-\lambda_2)h(\lambda_1)h(\lambda_2)d\lambda_1 d\lambda_2 \tag{3.7}$$

Taking the Fourier transform of both sides of Equation 3.7 yields:

$$\phi_\sigma(\omega) = \int_{-\infty}^{\infty} R_\sigma e^{-i\omega\tau}d\tau \tag{3.8a}$$

where

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$$\phi_\sigma(\omega) = \int_{-\infty}^{\infty} h(\lambda_1) \int_{-\infty}^{\infty} h(\lambda_2) \int_{-\infty}^{\infty} R_p(\tau+\lambda_1-\lambda_2)e^{-i\omega\tau}d\tau d\lambda_2 d\lambda_1 \qquad (3.8b)$$

The change of variable $\lambda = \tau + \lambda_1 - \lambda_2$ produces:

$$\phi_\sigma(\omega) = \int_{-\infty}^{\infty} h(\lambda_1)e^{i\omega\lambda_1} \int_{-\infty}^{\infty} h(\lambda_2)e^{-i\omega\lambda_2} \int_{-\infty}^{\infty} R_p e^{-i\omega\lambda}d\lambda \qquad (3.9)$$

The first two integrals are recognized as $H^*(\omega)$ and $H(\omega)$, respectively, and the third is $\phi_p(\omega)$ as given by Equation 3.5. The preceding equation can be written:

$$\phi_\sigma(\omega) = \phi_p(\omega)|H(\omega)|^2 \qquad (3.10)$$

This equation is the basic equation used in spectrum analysis. The procedure to calculate the output spectrum, in the case of single input, is now clear: The transfer function $H(\omega)$ is calculated first and then the square of its magnitude is multiplied by the assumed input power spectrum which, for the purpose of the analysis presented herein, is the von Karman spectrum function.

3.2 Output Spectrum of Multiple Inputs

Consider a linear system having multiple random inputs. In the case treated here, the wing of an airplane subjected to two-dimensional turbulence may be considered such a system. The variation of gust intensity in the spanwise direction produces different influence functions $h(\ell,y,t)$ at different sections of the wing. The total response then is the superposition of the response due to disturbances at each individual section. Hence, the response may be given by the following equation:

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$$\sigma(t) = \int_{-\ell}^{\ell} \int_{-\infty}^{\infty} p(y,\lambda)h(y,t-\lambda)d\lambda dy \tag{3.11}$$

where ℓ is the half length of the wing and $p(y,\lambda)$ is the input at time λ at a station y with width dy. Also, it is understood that the response is being computed at ℓ; hence, ℓ has been omitted from the parenthesis. An alternative form of Equation 3.11 is:

$$\sigma(t) = \int_{-\ell}^{\ell} \int_{-\infty}^{\infty} h(y,\lambda)p(y,t-\lambda)d\lambda dy \tag{3.12}$$

The correlation function of the output is:

$$R_\sigma(\tau) = \overline{\sigma(t)\sigma(t+\tau)}$$

$$= \int_{-\ell}^{\ell} \int_{-\ell}^{\ell} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(y_1,\lambda_1)h(y_2,\lambda_2)\overline{p(y_1,t-\lambda_1)p(y_2,t+\tau-\lambda_2)}$$

$$\times d\lambda_1 d\lambda_2 dy_1 dy_2 \tag{3.13}$$

where the overbar denotes the time average. Expressed in terms of the correlation function, R_p, of the input, Equation 3.14 becomes:

$$R_\sigma(\tau) = \int_{-\ell}^{\ell} \int_{-\ell}^{\ell} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(y_1,\lambda_1)h(y_2,\lambda_2)R_p(y_1,y_2,\tau+\lambda_1-\lambda_2)d\lambda_1 d\lambda_2 dy_1 dy_2 \tag{3.14}$$

Taking the Fourier transform and then using the change of variable $\lambda = \tau_T\lambda_1-\lambda_2$, Equation 3.14 becomes:

$$\phi_\sigma(\omega) = \int_{-\ell}^{\ell} \int_{-\ell}^{\ell} H^*(y_1,\omega)H(y_2,\omega)\phi_p(y_1,y_2,\omega)dy_1 dy_2 \tag{3.15}$$

where $\phi_p(y_1,y_2,\omega)$ is a cross spectrum function. From the definition of the cross spectrum, it can be seen [6] that the following relation holds:

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$$\phi_p(y_1,y_2,\omega) = \phi_p^*(y_2,y_1,\omega) \tag{3.16}$$

where "*" denotes the complex conjugate. Because of Equation 3.16 and because of the symmetry of the domain of integration and by changing the limits of integration, the integrand (with new limits of integration) in Equation 3.15 can be written as:

$$\begin{aligned}
&\phi_p(y_1,y_2,\omega)H(y_1,\omega)H^*(y_2,\omega) + \phi_p(y_2,y_1,\omega)H(y_2,\omega)H^*(y_1,\omega) \\
&\quad = \phi_p(y_1,y_2,\omega)H(y_1,\omega)H^*(y_2,\omega) + \phi_p^*(y_1,y_2,\omega)[H^*(y_2,\omega)H(y_1,\omega)]^* \\
&\quad = \phi_p(y_1y_2,\omega)H(y_1,\omega)H^*(y_2,\omega) + [\phi_p(y_1,y_2,\omega)H(y_1,\omega)H^*(y_2,\omega)]^* \\
&\quad = 2\mathrm{Re}[\phi_p(y_1,y_2,\omega)H(y_1,\omega)H^*(y_2,\omega)]
\end{aligned} \tag{3.17}$$

where Re[ ] is the real part of the quantity [ ]. Therefore, Equation 3.15 becomes:

$$\phi_\sigma(\omega) = \int_{-\ell}^{\ell}\int_{y_2}^{\ell} 2\mathrm{Re}[\phi_p(y_1,y_2,\omega)H(y_1,\omega)H^*(y_2,\omega)]dy_1dy_2 \tag{3.18}$$

By the assumption of isotropy of the atmospheric turbulence, the input spectrum $\phi_p(y_1,y_2,\omega)$ is a function only of the separation distance η, i.e., $\phi_p(y_1,y_2,\omega) = \phi_p(y_1-y_2,\omega)$. By this assumption and by making the change of variables $\eta = y_1 - y_2$ and $y = y_2$ and by interchanging the order of integration, Equation 3.18 becomes:

$$\phi_\sigma(\omega) = \int_0^{2\ell} \phi_p(\eta,\omega)2\mathrm{Re}[\int_{-\ell}^{\ell-\eta} H(y,\omega)H^*(y+\eta,\omega)dyd\eta] \tag{3.19}$$

This equation relates the output spectrum $\phi_\sigma(\omega)$ to the cross spectrum $\phi_p(\eta,\omega)$ of the inputs. In this study, $\phi_p(\eta,\omega)$ is the cross spectrum of atmospheric turbulence, and $H(y,\omega)$ is defined as the velocity of

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the right wing due to a unit sinusoidal gust of frequency ω located at y along the wing. Consequently, $\sigma(t)$, as defined in Equation 3.11, becomes the root mean square velocity of the right wing tip due to the gusts acting on the entire wing. Hence, $\phi_{\sigma}(\omega)$ is the spectrum of the velocity of the right wing tip caused by the vertical components of the gust.

CHAPTER IV

SPECTRUM OF WING TIP VELOCITY

In the Chapter I, two types of gusts were considered: one- and two-dimensional gusts. These two cases are illustrated in Figure 1, page 5. For one-dimensional gusts, the intensity of the gusts is the same along the span, and hence there is no tendency to roll. But for two-dimensional gusts, the intensities of the gusts at different points of the span are different. These variations in gust intensities along the span give rise to a net rolling moment, which then results in rolling motion.

Whether to consider a gust to be one- or two-dimensional depends on the relative sizes of the wing and the scale of turbulence. To elaborate on this statement, let us consider the gust to be composed of a number of sinusoidal gust components. The components with long wavelength tend to have uniform and small intensity along the span, and hence they produce no moment. The components with very small wavelength produce intensities along the wing that tend to cancel each other and again produce no tendency to roll. Figure 5 illustrates the effect of the components with long wavelength on the wing, and Figure 6 illustrates the effect of the components with short wavelength on the wing.

In contrast to the components with very long or very short wavelengths, the components with intermediate wavelength are found to be the major contributors to the rolling motion. These intermediate wavelengths are of the order from one to ten times the span of the wing.

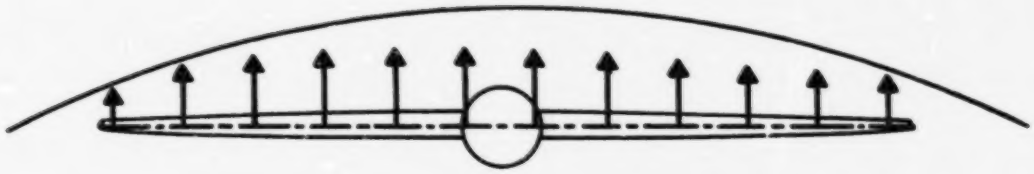

Figure 5 Effect of gust with long wavelength on the wing.

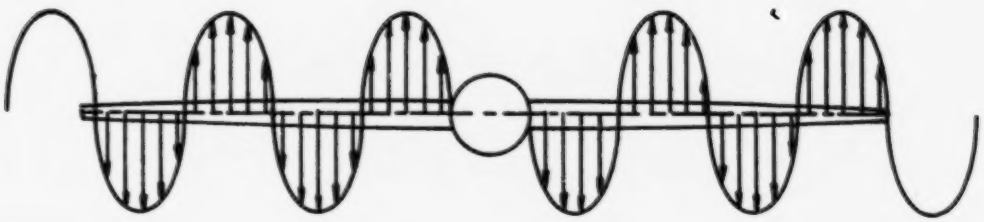

Figure 6 Effect of gust with short wavelength on the wing.

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The effect of a sinusoidal with wavelength equal to that of the span is illustrated in Figure 7.

The airplane considered in this study is a B-57 Cambera-type airplane which has a wing span of 2ℓ = 66 feet. Houbolt [7] recommends a length scale of atmospheric turbulence of L = 300 feet, which is about 1.8 times the span of the wing; a ratio that falls within the range of the intermediate components mentioned earlier. Hence, a two-dimensional spectrum analysis is required and the rolling motion should be taken into account.

The questions concerning the statistical description of the turbulence being settled (recall the assumptions given in the Chapter I and the discussion in the last few paragraphs) and following the outline

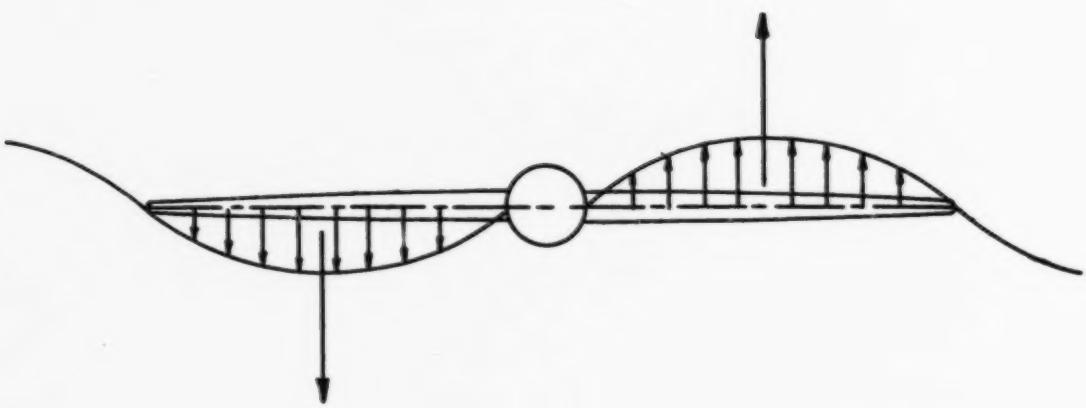

Figure 7 Effect of gust with wavelength equal to the length of the wing.

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given in the Chapter I there still remains the task of calculating the aerodynamic forces associated with the turbulent field, the determination of the transfer function from the equation of motion, and finally the calculation of the output spectrum.

Let us first consider the free vibration of a beam. Consider an element of length dy. This element undergoes a vertical motion and rotary motion about its center of mass and a shearing deformation. The free-body diagram and the geometry for the beam element are shown in Figure 8. The following quantities can be defined:

w = deflection of the centerline of the beam.

$\frac{dw}{dy}$ = slope of the centerline.

ψ = slope due to bending.

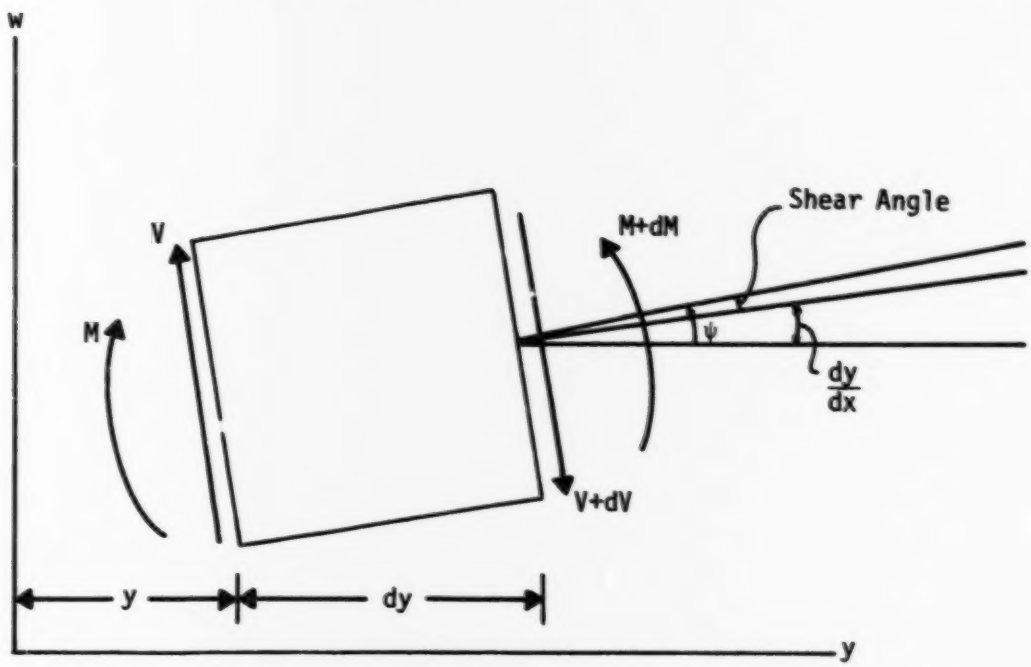

Figure 8 Free-body diagram of a beam segment.

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If there is no shear deformation, the perpendicular to the face of the cross section will coincide with the centerline of the beam. In case of shearing deformation, these lines will be distinct and the shear angle will be equal to $\psi - dw/dy$.

From flexure theory [8], we have these two elastic equations for the beam:

$$\psi - \frac{dw}{dy} = \frac{V}{k'AG} \tag{4.1}$$

and

$$\frac{d\psi}{dy} = \frac{M}{EI} \tag{4.2}$$

where V is the shear force, M is the bending moment, A is the cross-sectional area, k' is a factor depending on the shape of the cross section, G is the modulus of elasticity in shear, and EI is the bending stiffness. The equations of equilibrium of the moments and of the forces are given by:

$$J\ddot{\psi} = \frac{dM}{dy} - V \tag{4.3}$$

and

$$m\ddot{w} = -\frac{dV}{dy} \tag{4.4}$$

where J and m are the rotary inertia and mass of the beam per unit length, respectively.

Substituting Equations 4.1 and 4.2 into Equations 4.3 and 4.4, respectively, we get:

$$\frac{d}{dy}\left(EI\,\frac{d\psi}{dy}\right) + k'AG\left(\frac{dw}{dy} - \psi\right) - J\ddot{\psi} = 0 \tag{4.5}$$

and

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$$m\ddot{w} - \frac{d}{dy}\left\{k'AG\left(\frac{dw}{dy} - \psi\right)\right\} = 0 \qquad (4.6)$$

which are the coupled equations of free vibration for the beam in the general case. Theoretically, these equations can be solved simultaneously for arbitrary variations in elastic and inertial properties of the beam. In practice, closed solutions are difficult to obtain, except under certain simplifying assumptions. For example, if EI and A are constant, these two equations can be reduced, after eliminating ψ, to a single equation:

$$EI\,\frac{\partial^2 w}{\partial y^2} + m\,\frac{\partial^2 w}{\partial t^2} - \left(J + \frac{EIM}{k'AG}\right)\frac{\partial^2 w}{\partial y^2 \partial t^2} + \frac{Jm}{k'AG}\,\frac{\partial^4 w}{\partial t^4} = 0 \qquad (4.7)$$

The wing we are dealing with can be considered as a slender beam in which the cross-sectional dimensions are small in comparison with the length, and for which rotary inertial effects and shear deformations may be neglected [9]. In this case, the equations of motion, Equations 4.5 and 4.6, can be reduced to one equation as follows: Since the shear angle is zero (equivalently, the shear modulus G is infinite), ψ should be equal to dw/dy, also the term $J\ddot{\psi}$ in Equation 4.3 is neglected, then Equations 4.1 and 4.3 can be written as:

$$\psi = \frac{dw}{dy} \qquad (4.8)$$

and

$$V = \frac{dM}{dy} \qquad (4.9)$$

Substituting the value of V from Equation 4.9 into Equation 4.4 we get:

$$\frac{d^2M}{dy^2} = -m\ddot{w} \qquad (4.10)$$

The bending moment is related to the curvature by the flexure equation:

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$$M = EI \frac{d^2w}{dy^2} \tag{4.11}$$

Using this relation, Equation 4.10 becomes:

$$\frac{\partial^2}{\partial y^2}\left(EI \frac{\partial^2 w}{\partial y^2}\right) = -m\ddot{w} \tag{4.12}$$

As indiciated by Timoshinko, et al. [8], the error in calculating the natural frequencies of a beam with small cross-sectional dimension in comparison to its length using Equation 4.12 instead of Equations 4.5 and 4.6 is very small. Therefore, Equation 4.12 is adequate for calculating the natural frequencies of the beam, and it will be used in the analysis that will follow.

Consider a wing with varying mass and stiffness along its span. If a sinusoidal gust is acting on a spanwise element of length, Δ, centered about a point, y_1, then the following equation for the balance of forces holds:

$$\frac{\partial^2}{\partial y^2}\left(EI \frac{\partial^2 w}{\partial y^2}\right) = -m\ddot{w} - F_M + F_G\delta(y,y_1) \tag{4.13}$$

where

$$\frac{\partial^2}{\partial y^2}\left(EI \frac{\partial^2 w}{\partial y^2}\right)$$

is the force due to beam stiffness and $m\ddot{w}$ is the inertial force as defined in the preceding discussion. F_M is an aerodynamic force due to the motion of the wing and is given in [10] as:

$$F_M = \pi\rho b^2\ddot{w} + 2\pi\rho VbC(k)\dot{w} \tag{4.14}$$

where ρ is the density of the flight medium, b is the semi-chord of the wing, V is the mean flight speed, and C(k) is the Theodorsen function. The Theodorsen function is a function of the reduced frequency

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($k = \omega b/V$) of the motion. In terms of the Bessel functions, C(k) is:

$$C = \frac{J_1(J_1 + Y_0) + Y_1(Y_1 - J_0) - i(Y_1Y_0 + J_1J_0)}{(J_1 + Y_0)^2 + (Y_1 - J_0)^2} \tag{4.15}$$

All the functions appearing in Equation 4.15 are functions of the reduced frequency k. F_G is the force due to the vertical gust and is given [9] by:

$$F_G = 2\pi\rho V^2 b\left(\frac{u}{V}\right)K(k) \tag{4.16}$$

where K(k) is the Kussner function. K(k) can be expressed in terms of Theodorsen and Bessel functions:

$$K = C(J_0 - iJ_1) + iJ_1 \tag{4.17}$$

The function $\delta(y_2,y_1)$ which acts on F_G in Equation 4.13 selects the portion of the wing which is subjected to gust and is zero everywhere except between $y_1 - \Delta/2$ and $y_1 + \Delta/2$ where its value is unity.

Substituting the forces defined by Equations 4.14 and 4.16 into Equation 4.13 we obtain the following differential equation:

$$\frac{\partial^2}{\partial y^2}\left(EI\,\frac{\partial^2 w}{\partial y^2}\right) = -m\ddot{w} - \pi\rho b^2\ddot{w} - 2\pi\rho VbC(k)\dot{w} + 2\pi\rho V^2 bK(k)\,\frac{u}{V}\,\delta(y,y_1) \tag{4.18}$$

The boundary conditions for a free-body wing are:

$$w''(\ell,t) = w'''(\ell,t) = w''(-\ell,t) = w'''(-\ell,t) = 0 \tag{4.19}$$

These boundary conditions express the fact that there is no shear nor moment at the wing tips.

Equation 4.18 is a forced-vibration equation for the wing, the free-vibration equation is:

$$[EI(w''(y,t)]'' = -m\ddot{w}(y,t) \tag{4.20}$$

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Assuming that there is a solution for the free-vibration equation of the form:

$$w(y,t) = \phi(y)T(t) \tag{4.21}$$

we obtain, after substituting in Equation 4.20,

$$-\frac{1}{T}\frac{d^2T}{dt^2} = \frac{1}{m\phi}[EI\phi"]" \tag{4.22}$$

This equation is valid only if both sides are equal to some constant ω^2. Thus, Equation 4.22 is equivalent to the two equations:

$$\frac{d^2T}{dt^2} + \omega^2 T = 0 \tag{4.23}$$

$$[EI\phi"]" = m\omega^2\phi \tag{4.24}$$

The initial conditions for Equation 4.23 are:

$$T(0) = \dot{T}(0) = 0 \tag{4.25}$$

The boundary conditions of Equation 4.24 are:

$$\phi"'(\ell) = \phi"'(-\ell) = \phi"(\ell) = \phi"(-\ell) = 0 \tag{4.26}$$

which corresponds to the shear and bending moment being zero at the ends, as is the case for free ends.

Physically, ϕ represents the shape of a natural mode and ω is the vibration frequency corresponding to this mode. There is an infinite number of values of ω which satisfies Equation 4.20, and to each one of these values there corresponds a particular ϕ. Thus, the solution of Equation 4.20 can be expressed as:

$$w(y,t) = \sum_{i=1}^{\infty} \phi_i(y)T_i(t) \tag{4.27}$$

The natural modes, ϕ_i, are orthogonal [9] because of the choice of the boundary conditions. The orthogonality condition is given by:

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$$\int_{-\ell}^{\ell} [EI\phi_i"]"\phi_j dy = \omega_i^2 \int_{-\ell}^{\ell} m\phi_i\phi_j dy = \begin{cases} M_i\omega_i^2 & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases} \tag{4.28}$$

Substituting the value of w(y,t) from Equation 4.27 into Equation 4.18 and using the orthogonality property of the modes we get a system of linear differential equations in terms of the natural modes:

$$M_m\omega_m^2 T_m = -M_m\ddot{T}_m - \pi\rho b_R^2 \sum_{j=1}^{N} \bar{A}_{mj}\ddot{T}_j - 2\pi\rho V b_R C(k) \sum_{j=1}^{N} \bar{B}_{mj}\dot{T}_j$$

$$+ 2\pi\rho V^2 b_R K(k) \frac{u(y_1)}{V} \phi_m(y_1)a(y_1)\Delta \tag{4.29}$$

where $\bar{A}_{mj}$ and $\bar{B}_{mj}$ are aerodynamic cross terms given by:

$$\bar{A}_{mj} = \int_{-\ell}^{\ell} a^2\phi_m\phi_j dy \tag{4.30}$$

$$\bar{B}_{mj} = \int_{-\ell}^{\ell} a\phi_m\phi_j dy \tag{4.31}$$

The term a(y) is the semi-chord distribution defined as $a(y) = b(y)/b_R$. M_m is the generalized mass of the mode ϕ_m corresponding to natural frequency ω_m as defined by Equation 4.28. M_1 and M_2 are the mass and moment of inertia of the wing, respectively, and they are the generalized masses of the rigid-body vertical and rolling motions. These rigid-body modes have frequencies $\omega_1 = \omega_2 = 0$. In [1], the rigid-body rolling ϕ_2 was not included in the system of linear differential equations represented by Equation 4.29. As mentioned earlier, the restriction of the rigid-body motion to vertical motion only may cause an overestimation of the elastic vibrations of the wing. Hence, in the calculations

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that will follow, ϕ_2 will be included in the system of linear diffential equations so that the effect of rolling is accounted for in the solution of the system of equations.

Let us assume that the input and the output are sinusoidal having the forms:

$$\mu = \bar{\mu}\, e^{iks} \qquad \text{and} \qquad T_m = \bar{T}_m\, e^{iks} \tag{4.32}$$

where the variables k and s are defined as follows:

$$k = \frac{\omega b_R}{V} \qquad \text{and} \qquad s = \frac{Vt}{b_R} \tag{4.33}$$

Substituting the values of μ and T_m into Equation 4.29 and dividing throughout by $\pi\rho V^2 S\, e^{iks}$, we obtain a system of linear algebraic equations:

$$\mu_m \Omega_m^2 \bar{\xi}_m = k^2 \mu_m \bar{\xi}_m + k^2 \sum_{j=1}^{N} A_{mj} \bar{\xi}_j + 2ikC(k) \sum_{j=1}^{N} B_{mj} \bar{\xi}_j + \frac{2b_R K(k)}{S} \phi_m(y_1) a(y_1) \Delta \tag{4.34}$$

where

$$\Omega_m = \frac{\omega_m b_R}{V}$$

$$A_{mj} = \frac{b_R \bar{A}_{mj}}{S}$$

$$B_{mj} = \frac{b_R \bar{B}_{mj}}{S}$$

$$\mu_m = \frac{M_m}{\pi\rho b_R S}$$

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$$\bar{\xi}_m = \frac{\bar{T}_m}{b_R} \frac{V}{\bar{\mu}}$$

The solution $\bar{\xi}_m$ of the system of linear algebraic equations, Equation 4.34, represents the amplitude of the modal response of the deflection of the wing to a sinusoidal gust. If $\xi_m = \bar{\xi}_m e^{iks}$ is differentiated with respect to time, the amplitude of the wing tip velocity of the modal response is obtained:

$$\frac{d(\xi_m)}{dt} = ik \frac{T_m}{b_R} \frac{V}{\bar{\mu}} = i\omega \frac{T_m}{\bar{\mu}} \tag{4.35}$$

The transfer function, which is the velocity of the right wing tip velocity due to a sinusoidal gust located at y_1, is then:

$$H(y_1,\omega) = i\omega \sum_{j=3}^{N} \frac{\bar{T}_j(y_1,\omega)}{\bar{\mu}} \tag{4.36}$$

The summation in the preceding equation is carried over the bending modes only since the navigation system in the airplane can subtract the rigid-body motion from the turbulence data.

The numerical procedure for calculating the velocity spectrum of the wing tip is outlined in the following paragraphs (for full details see Reference [1]). The complete computer code with annotations is given in the appendix.

The numerical procedure is essentially divided into three subroutines. The free-vibration problem is solved in the first subroutine. The forced-vibration problem is solved in the second subroutine. Finally, the velocity spectrum of the right wing tip is calculated in the third subroutine.

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The first subroutine solves the free-vibration equation:

$$[EI\phi'']'' = m\omega^2\phi \qquad \text{for } -\ell \leq y \leq \ell \tag{4.37}$$

with boundary conditions

$$\phi'''(\ell) = \phi'''(-\ell) = \phi''(\ell) = \phi''(-\ell) = 0 \tag{4.38}$$

The coefficient m is the mass per unit length and EI is the bending stiffness of the wing. Both m and EI are a function of y. In [1], three different distributions of EI were considered. These are classified as: (1) standard wing, (2) stiff wing, and (3) flexible wing. The standard wing has a stiffness given by:

$$EI = 9 \times 10^8 \qquad \text{for } y \leq |11|$$

$$EI = 9 \times 10^7 \qquad \text{for } y \geq |11| \tag{4.39}$$

The above values were determined using a static analysis assuming a maximum loading on the B-57 wing and a load factor of 10 g. The fundamental frequency of this wing was found to be about 4 Hz.

The stiff wing has a bending stiffness given by:

$$EI = 3 \times 10^9 \qquad \text{for } y \leq |11|$$

$$EI = 3 \times 10^8 \qquad \text{for } y \geq |11| \tag{4.40}$$

These values were determined by a trial and error method so that the fundamental mode would have a frequency close to 7 Hz as determined from an in-flight experiment with the B-57 at NASA/Langley Research Center.

The flexible wing was considered for comparison purposes; it has a bending stiffness:

$$EI = 9 \times 10^6 \qquad \text{for } y \leq |11|$$

$$EI = 9 \times 10^5 \qquad \text{for } y \geq |11| \tag{4.41}$$

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The stiff wing is the most representative of a B-57 wing, and therefore it is used in the numerical calculations of this study. For comparison purposes, the standard wing is also considered.

EI and m are defined symmetrically in the domain $[-\ell,\ell]$ and so the differential equation is either symmetric or antisymmetric. Because of this, the problem needs to be solved only for $0 \leq y \leq \ell$. In this case, the boundary conditions must be defined at the midspan of the wing. The conditions at the origin for a symmetric function are:

$$\phi'(0) = \phi'''(0) = 0 \tag{4.42}$$

and the conditions for an antisymmetric function are:

$$\phi(0) = \phi''(0) = 0 \tag{4.43}$$

An initial value of ω is assumed, and then the fourth-order differential equation is changed to a system of four first-order equations. A shooting method is used to estimate the complete set of initial conditions, after which a Runge-Kutta scheme is used to determine the solution of the initial value problem. The value of the solution of the initial value problem at $y = \ell$ is compared with the boundary conditions at $y = \ell$ for the two-point boundary value problem. The estimate of the complete set of initial conditions is improved. The process is repeated until the initial value problem has the correct value for the boundary condition at $y = \ell$. The system of four first-order equations:

$$\frac{d}{dy}\begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\end{pmatrix} = \begin{pmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ \frac{m\omega^2}{EI} & 0 & \frac{-2EI'}{EI} & \frac{-EI''}{EI}\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\end{pmatrix} \tag{4.44}$$

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has a vector base, $\{\bar{\psi}\}_{i=1}^{4}$, of its solution. Each solution:

$$\bar{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix} \tag{4.45}$$

of Equation 4.43 is a linear combination of the base solution, i.e.,

$$\bar{\phi} = \sum_{i=1}^{4} \bar{\psi}_i C_i$$

For $\{\bar{\psi}_i\}_{i=1}^{4}$ to be a complete base, there should be four linearly independent intial conditions. A simple choice of linearly independent initial conditions is:

$$\bar{\psi}_1(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} ; \bar{\psi}_2(0) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} ; \bar{\psi}_3(0) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} ; \bar{\psi}_4(0) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{4.46}$$

The boundary conditions are used to determine the constants $\{C_i\}_{i=1}^{4}$. In the case of the even mode, we have the system of equations:

$$\begin{pmatrix} \psi_{12}(0) & \psi_{22}(0) & \psi_{32}(0) & \psi_{42}(0) \\ \psi_{14}(0) & \psi_{24}(0) & \psi_{34}(0) & \psi_{44}(0) \\ \psi_{13}(\ell) & \psi_{23}(\ell) & \psi_{33}(\ell) & \psi_{43}(\ell) \\ \psi_{14}(\ell) & \psi_{24}(\ell) & \psi_{34}(\ell) & \psi_{44}(\ell) \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{4.47}$$

To have a nontrivial solution of the above equation, the determinant of the coefficient matrix must vanish. This determinant:

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$$D(\omega) = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \psi_{13}(\ell) & \psi_{23}(\ell) & \psi_{33}(\ell) & \psi_{43}(\ell) \\ \psi_{14}(\ell) & \psi_{24}(\ell) & \psi_{34}(\ell) & \psi_{44}(\ell) \end{vmatrix} = \begin{vmatrix} \psi_{13}(\ell) & \psi_{43}(\ell) \\ \psi_{14}(\ell) & \psi_{44}(\ell) \end{vmatrix} \qquad (4.48)$$

is a function of ω, the unspecified parameter of the differential equation. This characteristic determinant vanishes when the correct values of the natural frequencies, ω_i, of the wing are found.

Now, we summarize the preceding numerical procedure. First, the eigenvalue is estimated; then the fundamental solutions are determined by a Runge-Kutta/Fehlberg order seven scheme. The value of the characteristic determinant is calculated from the fundamental solutions. A search routine checks if the natural frequency is bracketed between the current estimate and the previous estimate. In this case, the program is directed to a bisection routine to improve brackets or continues, taking another step along the frequency domain and using this as its next estimate of the frequency. After the frequency has been determined, the natural mode is normalized by a unit displacement at the right wing tip. The new mode is integrated with previous modes determined to calculate the aerodynamic cross terms. The program then steps along the frequency domain for its first and second estimates of the next frequency.

The second subroutine of the program solves the system of linear algebraic equations, Equation 4.34. This system is solved by Gaussian elimination for 38 gust locations evenly spaced along the wing.

The third subroutine calculates the output power spectrum. It integrates Equation 3.19 numerically using the trapezoidal rule:

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$$\phi_\sigma(n,\omega) = \phi_p(0,\omega) \sum_{j=1}^{N} H_j H_j^* + \sum_{j=1}^{N-1} \phi_p(j\Delta,\omega) 2\mathrm{Re}\left[\sum_{i=1}^{N-j} H_i H_j^*\right] \tag{4.49}$$

where N is the number of gust stations and Δ is the gust station width. The spectrum program determines the wing tip velocity spectrum; therefore, H_i must represent the velocity at the right wing tip due to a sinusoidal gust at station i. The frequency response function is defined as:

$$H_i = \frac{i\omega}{\mu} \sum_{j=1}^{NM} \bar{T}_j \tag{4.50}$$

where NM is the number of elastic modes considered. The summation includes the response of only the elastic modes because, as mentioned earlier, the navigation system located at the airplane's center of gravity allows subtraction of the rigid body motions from the turbulence data taken at the wing tip.

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CHAPTER V

DISCUSSION AND CONCLUSIONS

In the spectrum analysis section, a mathematical treatment of stationary processes was shown. In order to apply these techniques to problems dealing with atmospheric turbulence, the turbulence was assumed to be homogeneous. In general, most types of turbulence tend to be homogeneous, except those at low altitudes which may be influenced by the configuration of the ground and those in thunderstorms.

The assumption of isotropy simplifies the expressions of two-dimensional correlation functions because in this case a correlation function can be expressed as one-dimensional correlation function of the separation distance. For sufficiently short wavelengths, all turbulence is isotropic [10], but for long wavelengths it is isotropic only if it is homogeneous.

Taylor's hypothesis permits time correlation functions to be expressed as space correlation functions. This is so because the gust intensities remain the same until the airplane traverses this region of turbulence. The speed of the airplane plays an important role in validating this hypothesis. At very low speeds, Taylor's hypothesis becomes less valid and the results calculated may be less accurate.

A quantity of great importance is the turbulence length scale L. Roughly speaking, L is a measure of the minimum separation distance for which there is no correlation between two velocity components of the gust. Many factors, such as weather conditions, altitude, and ground configurations (for low altitudes) play a role in determining L. The

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estimates for L range between a few tens of feet to thousands of feet. In this study, two turbulence length scales of 132 feet and 2,112 feet were considered. Figures 9 and 10 show the turbulence spectrum and the wing tip velocity spectrum of a standard wing and a stiff wing for these two turbulence scales.

The relative errors in the atmospheric spectrum due to wing tip vibration are shown in Figures 11 through 14. These were calculated using the following reasoning: The wind velocity measured at the wing tip is an apparent velocity since in reality it is the sum of the wing tip velocity and the true velocity of the wind. This can be expressed by the equation:

$$U_m = U_p + U_r \tag{5.1}$$

where U_m is the measured velocity, U_p is the true velocity of the turbulence, and U_r is the wing tip velocity. From Equation 5.1 the correlation function of the measured velocity can be expressed by the following equation:

$$\begin{aligned} R_m(\tau) &= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} U_m(t)U_m(t+\tau)dt \\ &= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} [U_\sigma(t)U_\sigma(t+\tau) + U_p(t)U_p(t+\tau) + U_\sigma(t)U_p(t+\tau) \\ &\quad + U_p(t)U_\sigma(t+\tau)]dt \\ &= R_\sigma(\tau) + R_p(\tau) + R_{\sigma p}(\tau) + R_{p\sigma}(\tau) \end{aligned} \tag{5.2}$$

Taking the Fourier transform of the above equation we get:

$$\phi_m(\omega) = \phi_\sigma(\omega) + \phi_p(\omega) + \phi_{\sigma p}(\omega) + \phi_{p\sigma}(\omega) \tag{5.3}$$

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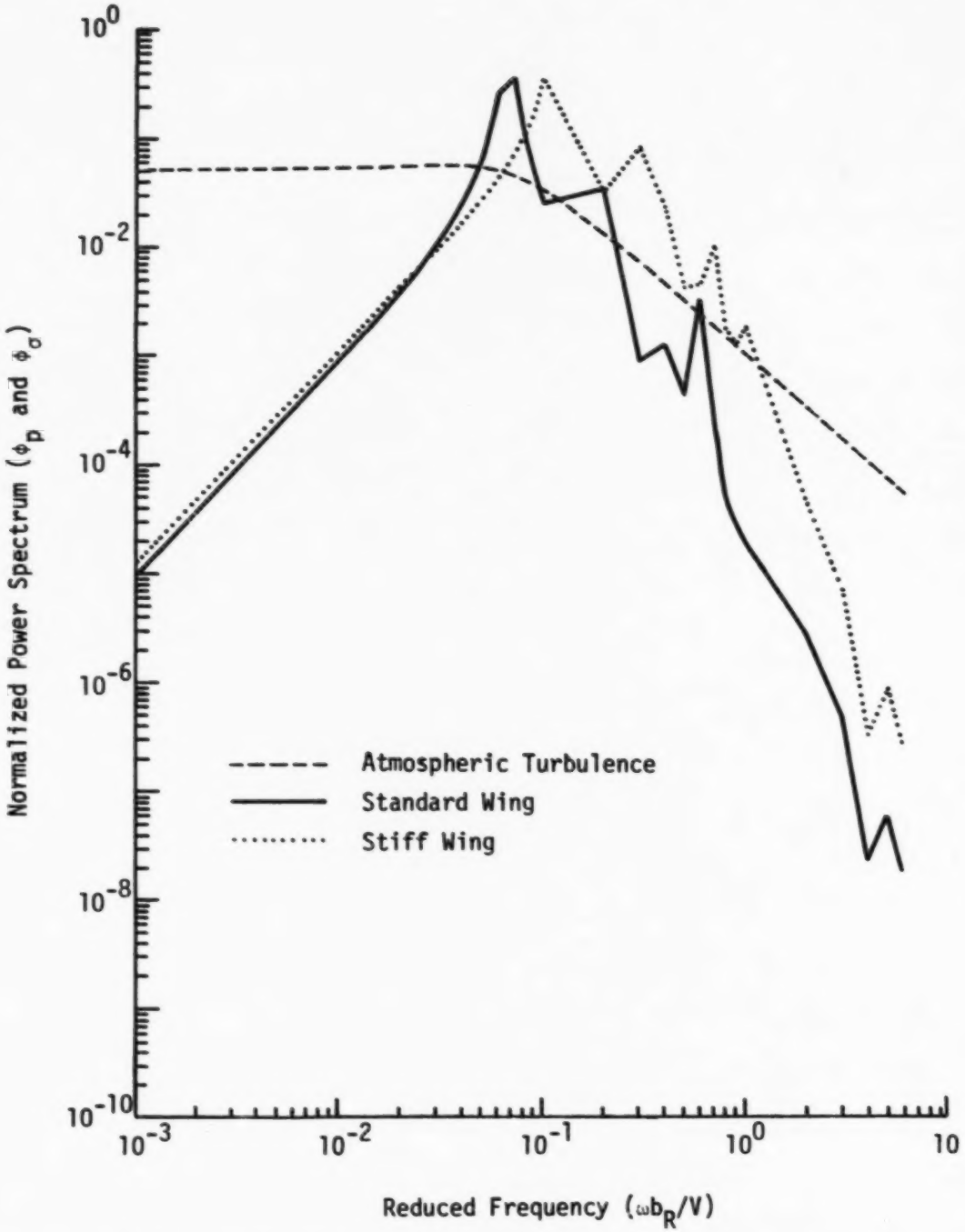

Figure 9 Spectra of wing tip velocity for standard and stiff wings in atmospheric turbulence having spectrum illustrated (length scale = 132 feet).

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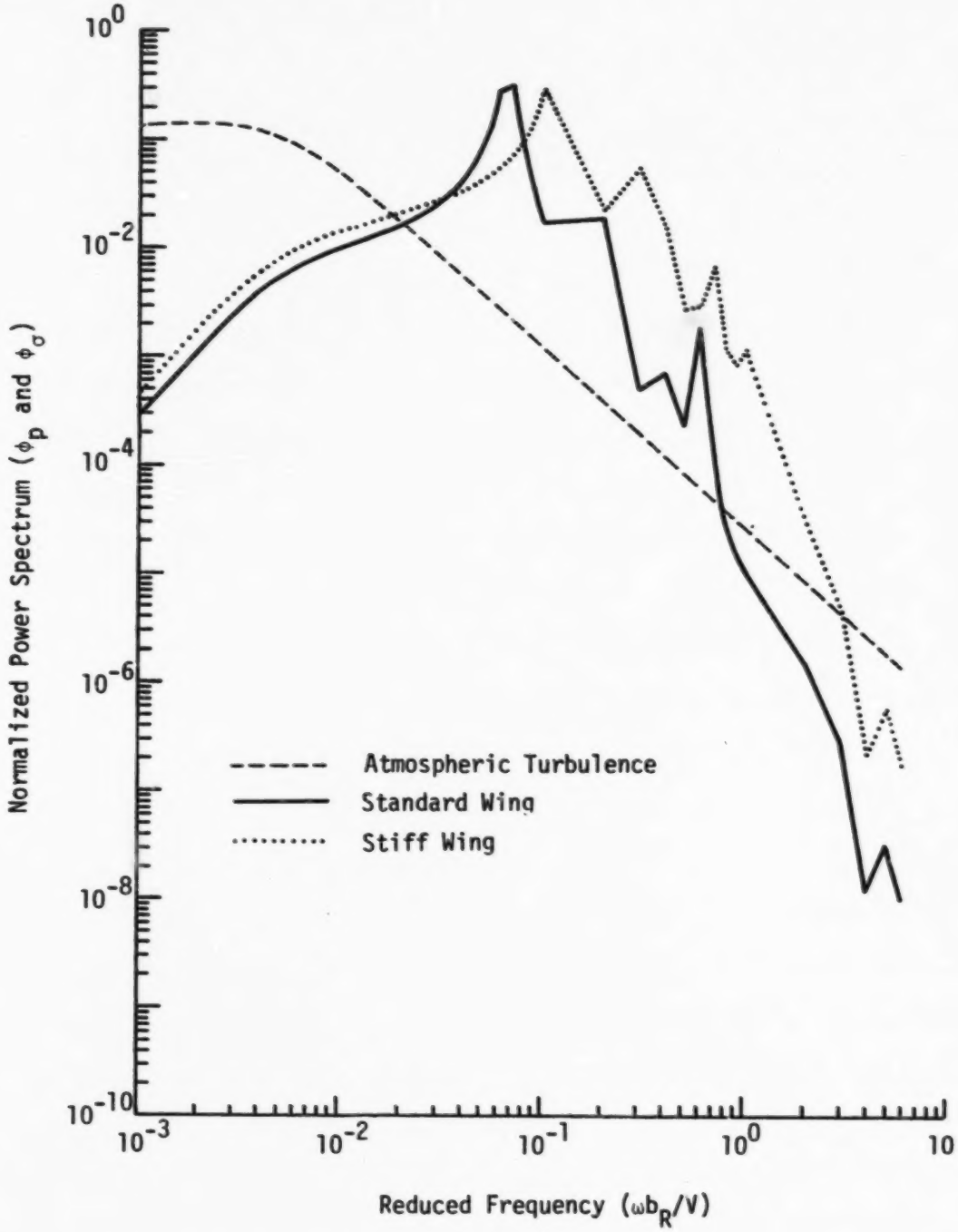

Figure 10 Spectra of wing tip velocity for standard and stiff wings in atmospheric turbulence having spectrum illustrated (length scale = 2,112 feet).

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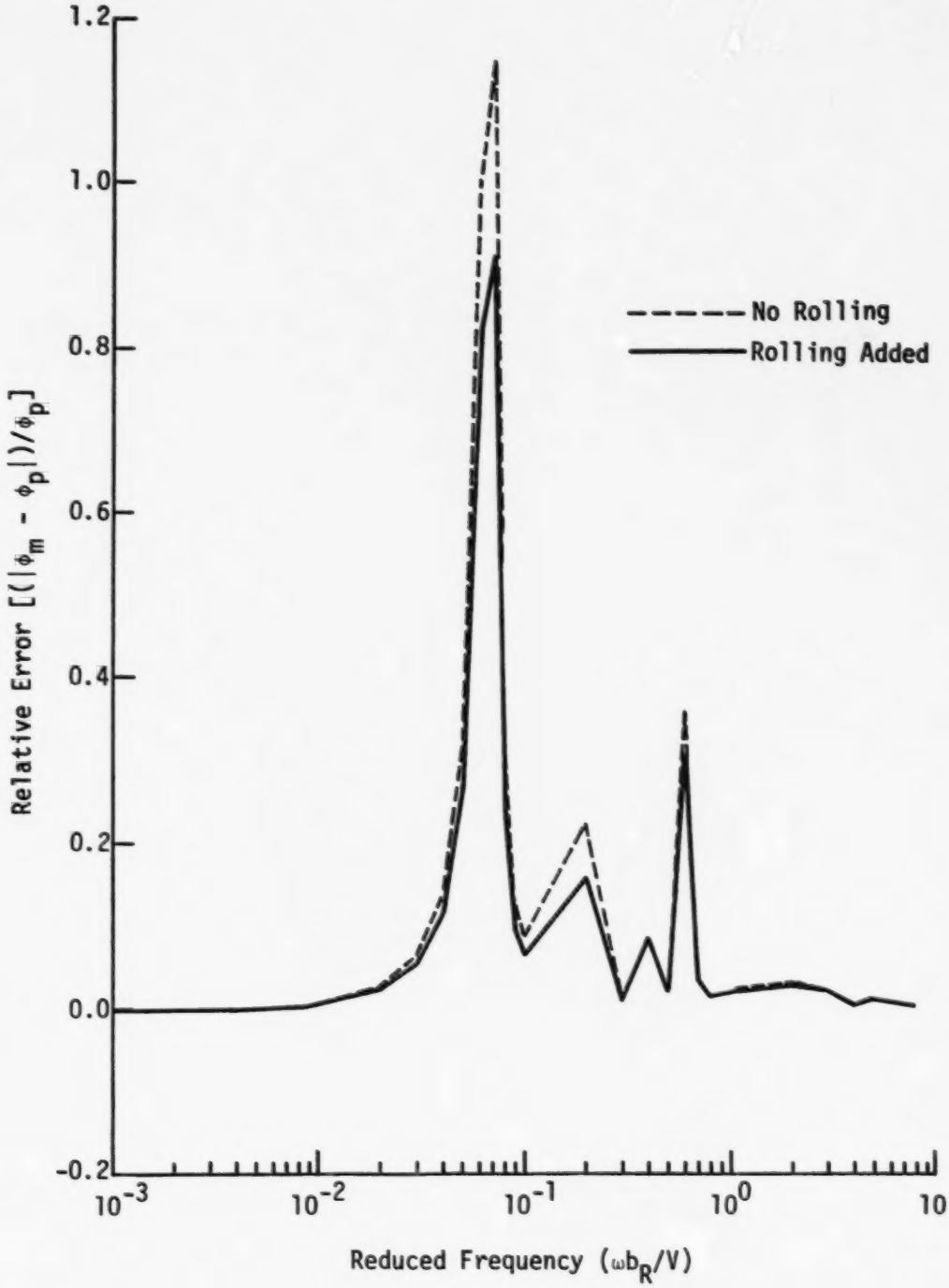

Figure 11 Relative error in measuring turbulence with length scale 132 ft for standard wing.

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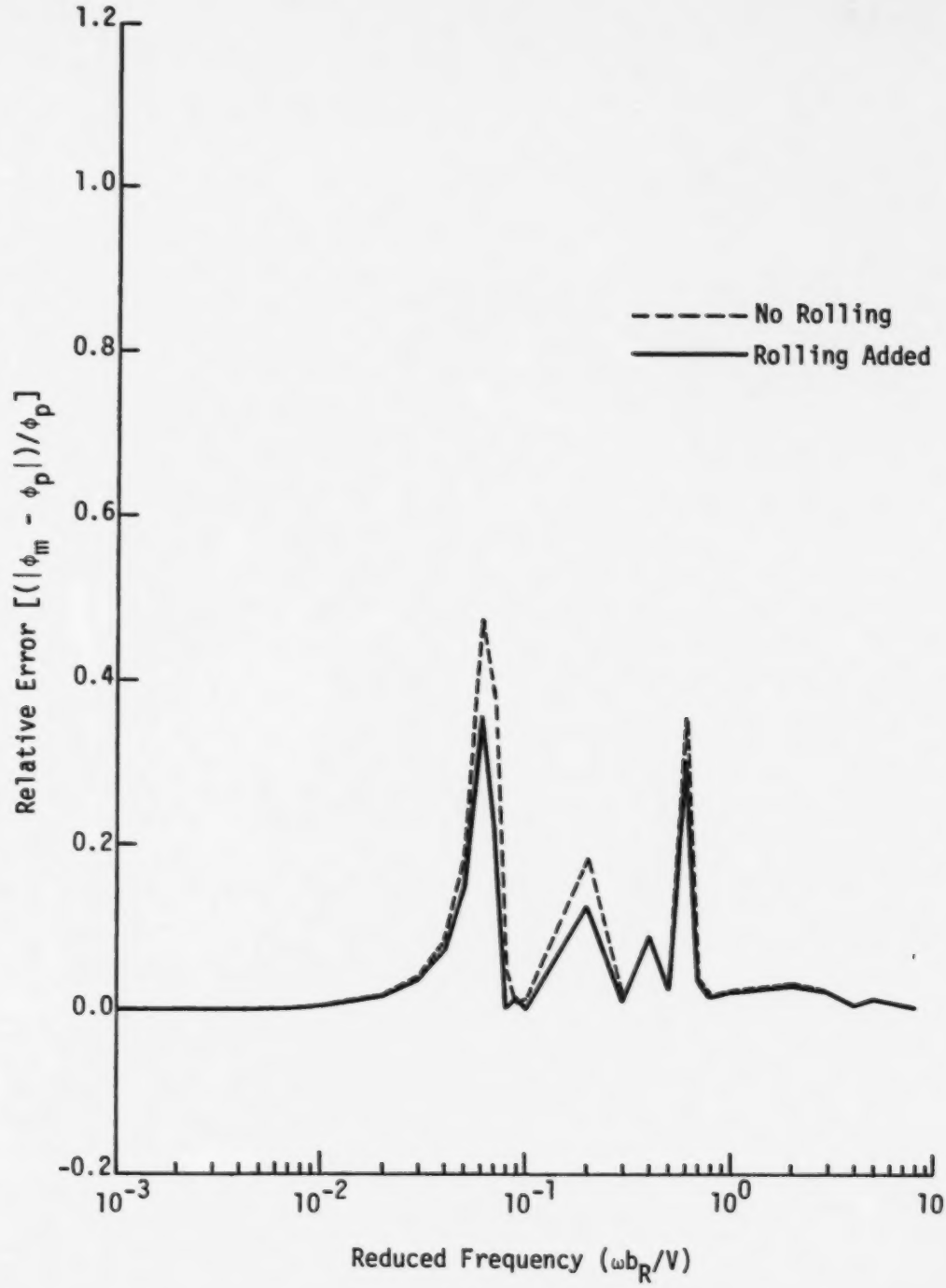

Figure 12 Relative error in measuring turbulence with length scale 2,112 ft for standard wing.

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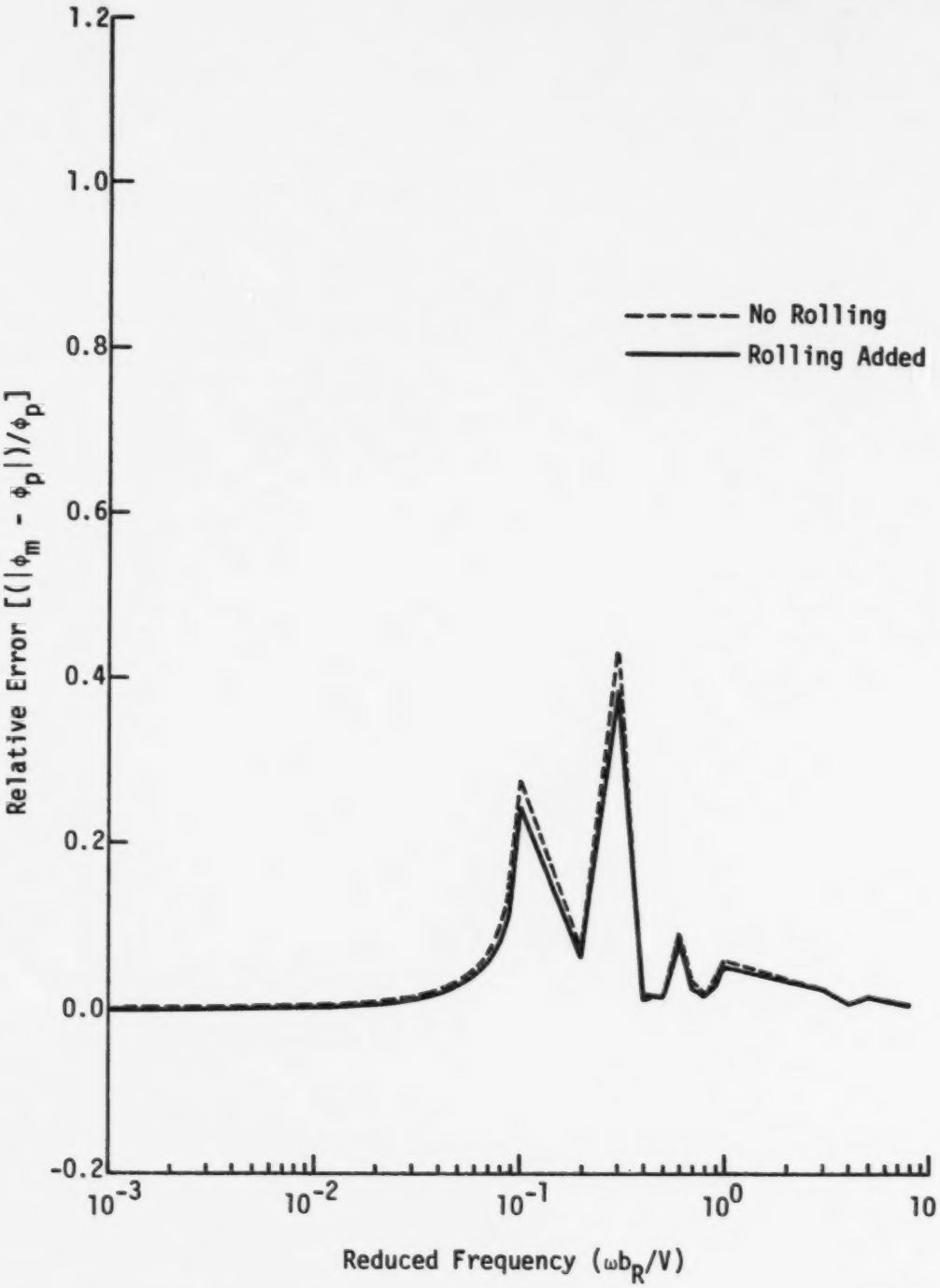

Figure 13 Relative error in measuring turbulence with length scale 132 ft for stiff wing.

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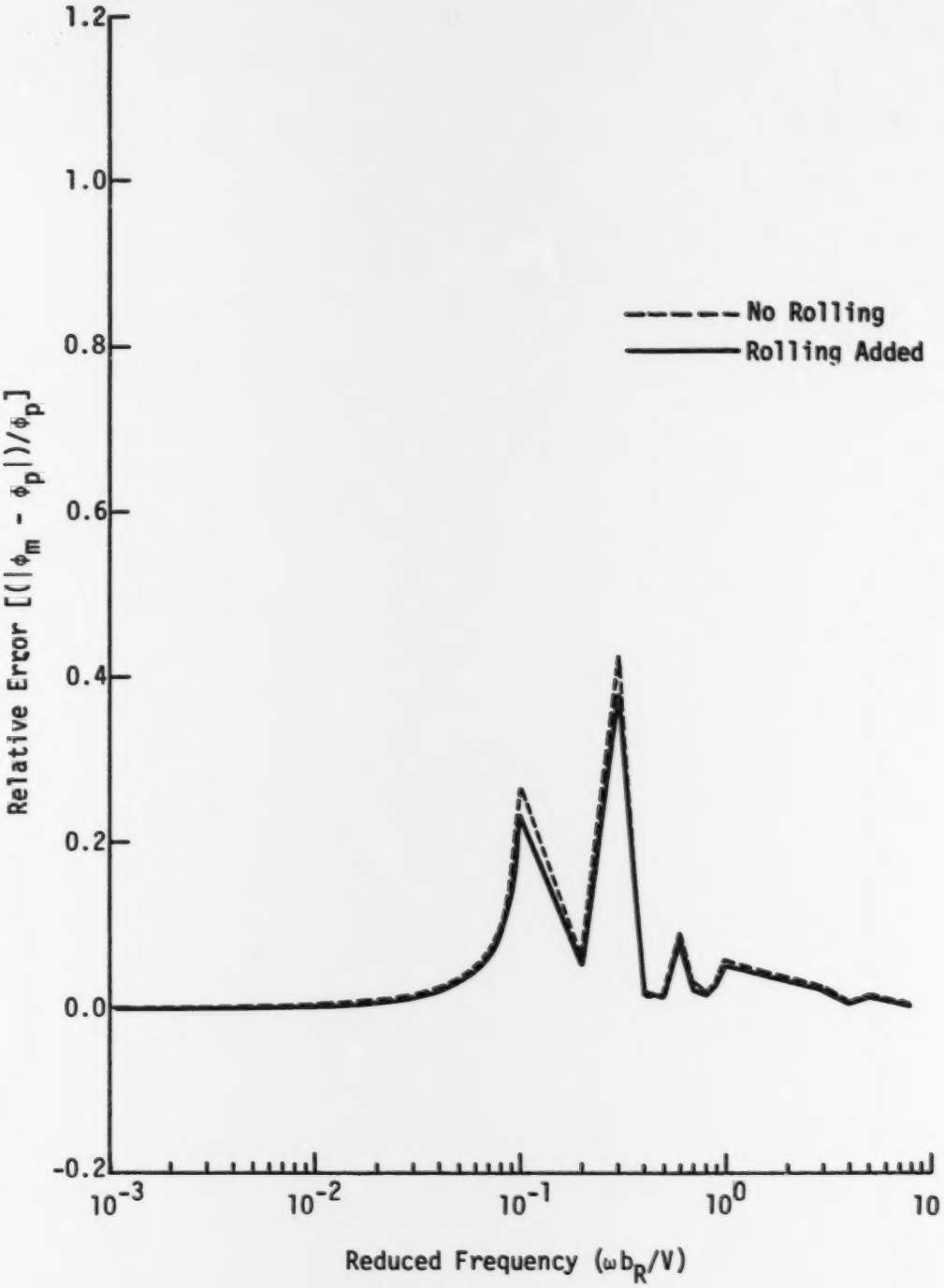

Figure 14 Relative error in measuring turbulence with length scale 2,112 ft for stiff wing.

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Using the relation $\phi_{\sigma p} = \phi^*_{p\sigma}$, Equation 5.3 becomes:

$$\phi_m = \phi_\sigma + \phi_p + 2Re\phi_{\sigma p} \tag{5.4}$$

where ϕ_p is the turbulence spectrum at the wing tip and ϕ_r is the power spectrum of the wing tip velocity. The equation for $\phi_{\sigma p}$ can be written as:

$$\phi_{\sigma p} = \int_0^{2\ell} \phi_p(\eta) H(\ell - \eta) d\eta \tag{5.5}$$

The relative error due to the wing tip velocity can be written as:

$$E = \frac{|\phi_m - \phi_p|}{\phi_p} = \frac{|\phi_\sigma + 2Re\phi_{\sigma p}|}{\phi_p} \tag{5.6}$$

Figure 11 shows the relative error as a function of frequency for a standard wing and turbulence scale of 132 feet for two cases: (1) the rigid-body motion is restricted to vertical motion and (2) the rigid-body rolling is taken into account. This wing shows a large relative error close to its fundamental frequency. This error reaches a maximum of about 115 percent when the wing is restricted to vertical motion. This maximum is reduced to about 94 percent when the rolling motion is taken into account. The true error is believed to lie within these limits (see Chapter II, page 5). The range in which this maximum occurs falls within the frequency range which contains significant turbulence energy. For a turbulence length scale equal to 2,112 feet, the maximum relative error (see Figure 12) for the standard wing is reduced to about 50 percent when rolling is not considered and to about 38 percent when rolling is added. But, this time the maximum relative error falls within a range which is out of the frequency range that contributes sig-

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nificant turbulence energy. The stiff wing shows a smaller relative error than the standard wing and the range of the maximum error is slightly shifted to higher frequency. The addition of rolling motion reduces this maximum from about 50 to 36 percent. For smaller turbulence scales, this maximum error is very close to the range of frequencies which contribute significant turbulence energy. But, for larger scales this error is outside the range of frequencies of interest.

From the preceding discussion it can be seen that a stiff wing is the best wing to measure turbulence and but even then in order to measure accurately the whole range of the atmospheric spectrum depending on the length scale, accelerometers mounted on the wing tips are required.

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APPENDIX

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APPENDIX

DOCUMENTATION OF THE COMPUTER PROGRAM VIB.FOR

The purpose of this appendix is to document the computer program VIB.FOR used in [1] and, with modifications, in the text. Flowcharts illustrating the complete computer code are given in [1]. An explanation of the subroutines and parameters appearing in the program will follow.

VIB.FOR is essentially divided into three subprograms: (1) the free-vibration subroutine, D1, (2) the forced-vibration subroutine, D2, and (3) the spectrum-analysis subroutine, D3. Each of D1, D2, and D3 has a number of subroutines and functions arranged in the following order:

I. SUBROUTINE D1

1. Subroutine SZERO(W,H,N,NC,NN)
2. Subroutine RUN(W,F,EL,EU,NC,NN)
3. Subroutine BISEC(X1,F1,X2,F2,NC,NN)
4. Subroutine FUNEV(K,X,Y,F)
5. Function SEI(X)
6. Subroutine SECANT(X1,F1,X2,F2,NC,NN)
7. Subroutine DMODE(YY,NN)
8. Function RM(X)
9. Function SIMP(Y,H,N)
10. Subroutine COF(N,YY,W2,NN,I1)
11. Subroutine STRK(NN)
12. Subroutine RK7(NS,NN,EL,EU)

II. SUBROUTINE D2

1. Subroutine DO(NN,N2)
2. Subroutine COEF(RK,X,DEL,N2M,NN)
3. Function RPH(I,Y,N)
4. Function RA(Y)
5. Function RY1(X)
6. Complex Function CC(RK)
7. Complex Function RKK(RK)
8. Function RJ1(X)
9. Function RY0(X)
10. Subroutine GAUSS(N,N2)
11. Subroutine BACKS(N1,N2)

III. SUBROUTINE D3

1. Subroutine SPEC(RK,RR,TL,N2,IC,RP)
2. Function TSPEC(SS,RNU,U,TL)
3. Subroutine COEF1
4. Function BSL1(Z)
5. Function POLY(A,N,Z)
6. Function BSL2(Z)
7. Subroutine COEF2

A.1 Subroutine D1

In this subroutine the first four frequencies and elastic modes are calculated. Frequencies are calculated by starting with an initial guess after which the differential equation representing the free vibration is solved. From this solution the value of the characteristic

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determinant is calculated. The first frequency is incremented and the differential equation is solved for this new value of the frequency. Then the value of the characteristic determinant is calculated and is compared with the previously calculated value. If these two values have the same sign, the frequency is incremented again and the differential equation is solved and the characteristic determinant is calculated. This process is done repeatedly until the value of the characteristic determinant changes sign, after which a bisection method and a secant method are used to approximate the frequency that makes the characteristic determinant vanish. The mode shape corresponding to this frequency is calculated, and then the generalized mass and the aerodynamic cross terms of all previously determined modes are calculated. The same procedure is used until the first four natural frequencies and the corresponding elastic modes are determined together with all generalized masses and aerodynamic cross terms.

Parameters. NC is the number of the mode to be calculated (the first value of NC is two because model number one is considered to be the rigid body translation mode which has zero frequency and a constant normalized value equal to unity. Modes number two, three, four, and five are the first four elastic modes. Mode number six is taken in this program to be the rigid body rolling mode). H is the step size in the frequency domain. W is the initial guess of the natural frequency. N = 4 is the number of elastic modes to be calculated. NN = 151 is the number of nodes on a semi-span at which the values of the modes are calculated.

```
      COMMON/EDAT/EEE
      CALL D1
      CALL D2
      CALL D3
      STOP
      END
      SUBROUTINE D1
C*** THIS PROGRAM IS USED FOR DETERMING WING'S NATURAL
C     FREQUENCY AND MODE.
C
C     IT USES A BISECTION LIKE METHOD TO IMPROVE THE GUESS
C     ON THE FREQUENCY AND MAKES THE CHARACTERISTIC DETERMINANT
C     VANISH.
C
C     RUNGE-KUTTA FEHLBERG 7&8 IS USED TO DETERMINE SOLUTION
C     TO DIFFERENTIAL EQN IT HAS VAIRABLE SIZING BETWEEN
C     FIXED NODES.
C
C     SIMPSONS METHOD IS USED TO INTEGRATE FOR THE AREODYNAMIC
C     CROSS TERMS AND GENERALIZED MASS.
C
C     THE PROGRAM MAYBE ADOPTED BY CHANGING FUN...SEI
C     (THE FUNCTION DESCRIBING THE DISTRIBUTION OF THE PRODUCT
C     YOUNG'S ELASTICITY AND SECTIONAL MASS MOMENT, IF SEI IS TO
C     HAVE DERIVATIVES UP TO THE SECOND ORDER FUN...FUNEV NEEDS TO BE
C     CHANGED TO INCLUDE THEM IN THE DIFFERENTIAL EQN.  THE WING
C     PLAN ARRAY, RA, WILL NEED TO BE CHANGED.
C***NC=THE NUMBER OF MODE TO START CALCULATING
C***W=THE FRIST GUESS OF THE NATURAL FREQUENCY OF THE NC MODE
C***H=STEP SIZE FOR SEARCH ROUTINE
C***N=TOTAL NUMBER OF MODES TO CALCULATE
C***NN=NUMBER OF NODES TO CALCULATE MODE ON
      REAL*8 W,H
      NC=2
      H=1.5D0
      W=0.1D0
      N=4
      NN=151
      CALL STRK(NN)
      CALL SZERO(W,H,N,NC,NN)
      RETURN
      END
```

A.2 Subroutine SZERO(W,H,N,NC,NN)

Description. This subroutine starts with an initial guess of the natural frequency and calls subroutine FUN which fixes the initial values of the differential equation. Then FUN calls subroutine RK7 which solves the differential equation for the given frequency using Runge-Kutta-Fehlberg method of order seven. After obtaining the solution of the differential equation, FUN calculates the value of the characteristic determinant and returns it to SZERO. Then, SZERO steps along the frequency domain and repeats the above procedure to calculate a second value of the characteristic determinant. SZERO checks for a change in sign of the calculated values of the determinant. If there is no change of sign, it steps again along the frequency domain and calculates a new value of the characteristic determinant, and then it checks

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for a change in sign of the last calculated values. If there is a change in sign, SZERO calls subroutine BISECTION to approximate the frequency that makes the characteristic determinant vanish. Then, SZERO calls subroutine SECANT to improve the approximation found by BISECTION, after which it calls subroutine DMODE to calculate the normalized values of the mode at 151 points of the semi-span. Finally, SZERO calls subroutine COF to calculate the generalized mass of the mode and the aerodynamic cross terms of the mode with the other modes. The frequency which was calculated last is used as an initial guess for the next frequency and the whole process is repeated until the first four frequencies are calculated.

Parameters. I is a counter which gives the number of calculated modes. EL and EU are error bounds for Runge-Kutta method. W1 and W2 are two consecutive values in the frequency domain. F1 and F2 are the corresponding values of the characteristic determinant.

```
      SUBROUTINE SZERO(W,H,N,NC,NN)
C     THIS SUBROUTINE STEPS ALONG THE FREQUENCY DOMAIN UNTILL
C     IT BRACKETS THE NATURAL FREQUENCY AND THEN CALLS A BISECTION
C     ROUTINE AND A SECANT ROUTINE TO IMPROVE THE WIDTH OF THE BRACKETS.
C     THE SUBROUTINE FINALLY CALLS SUB...DMODE TO DETERMINE THE
C     MODE, AND THEN CALLS COEF TO SET UP THE INTERGRATIONS OF MODES
C     AND THEIR PRODUCTS.
C**** EL & EU=ERROR BOUNDS FOR RUNGE KUTTA
C**** W1 & W2=BRACKETS FOR FREQUENCY AND STEPS FOR THE SEARCH ROUTINE
C**** F1 & F2=VALUES OF THE CHARACTERISTIC DETERMINTE FOR W1 & W2
C**** N=NUMBER OF EIGENVALUES TO BE SEARCHED
C**** W=GUESS FOR EIGEN VALUE
C**** H=STEP SIZE FOR SEARCH
C**** NC=NUMBER OF THE FRIST MODE TO CALCULATE
C**** NN=NUMBER OF NODES TO CALCULATE MODES ON
C**** I=NUMBER OF MODES CALCULATED DURING PROCESS
      REAL*8 W,H,W1,W2,F1,F2,EL,EU,Y,X
      COMMON/FACT/Y(8,151),X(151),GM(6)
      DIMENSION YY(151),XX(151)
      I=0
      W1=W
      EL=.0001D0
      EU=.001D0
  103 CALL FUN(W1,F1,EL,EU,NC,NN)
  101 W2=W1+H
      CALL FUN(W2,F2,EL,EU,NC,NN)
      IF(F1*F2.LT.0.0) GO TO 102
      W1=W2
      F1=F2
      GO TO 101
  102 CONTINUE
      CALL BISEC(W1,F1,W2,F2,NC,NN)
      CALL SECANT(W1,F1,W2,F2,NC,NN)
      CALL DMODE(YY,NN)
      I=I+1
      WS=W2
      CALL COF(NC,YY,WS,NN,I)
      NC=NC+1
```


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```
      IF(I.GE.N)GO TO 104
      W1=W2+H
      GO TO 103
104   DO 8710 I=1,NN
8710  YY(I)=(I-1)/150.
      CALL  COF(6,YY,0.,NN,5)
      RETURN
      END
```

A.3 Subroutine FUN(W,F,EL,EU,NC,NN)

Description. This subroutine estimates the initial conditions for the even and odd modes and then calls RK7 to solve the differential equation. Finally, it determines the characteristic determinant and returns it to SZERO.

Parameters. EL and EU are error bounds for Runge-Kutta method. NC is the number of mode being determined. W is the frequency. F is the value of the characteristic determinant. Y is the matrix of the solution of the system of differential equations solved in RK7.

```
      SUBROUTINE FUN(W,F,EL,EU,NC,NN)
C     THIS SUBROUTINE FIXES THE INITIAL VALUE FOR THE SOLUTION
C     AND THEN CALLS SUB.,RK7(RUNGE KUTTA ROUTINE) TO DETERMINE SOLUTION
C     AFTER WHICH THIS SUB CALCULATES THE VALUE OF THE CHARACTERISTIC
C     DETERMINANT.
C*** EL & EU =ERROR BOUNDS FOR RUNGE KUTTA
C*** NC=WHICH MODE WORKING ON DETERMING
C*** Y=FUNDAMENTAL SOLUTIONS TO DIFFERENTIAL EQN OUTPUT FROM RK7
C*** F=VALUE OF CHARACTERISTIC DETERMINTE
      REAL*8 RK,W,F,EL,EU,Y,X
      COMMON/FACT/Y(8,151),X(151)
      COMMON/EGNVL/RK
      RK=W
C INITIAL CONDITION TEST FOR EVEN OR ODD MODE
      DO 100 I=1,8
 100  Y(I,1)=0.D0
      IF(NC.EQ.2.OR.NC.EQ.4) GO TO 101
C INITIAL CONDITION FOR ODD MODE
      Y(2,1)=1.D0
      Y(8,1)=1.D0
      GO TO 500
C INITIAL CONDITION FOR EVEN MODE
 101  Y(1,1)=1.D0
      Y(7,1)=1.D0
 500  CALL RK7(8,NN,EL,EU)
      F=Y(3,NN)*Y(8,NN)-Y(7,NN)*Y(4,NN)
      RETURN
      END
```

A.4 Subroutine BISEC(X1,F1,X2,F2,NC,NN)

Description. This subroutine approximates the frequency after finding an upper and a lower bound.

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Parameters. X1 is the lower bound of the frequency and X2 is the upper bound. F1 and F2 are the values of the characteristic determinant. NC is the mode number of the frequency being determined.

```
      SUBROUTINE BISEC(X1,F1,X2,F2,NC,NN)
      REAL*8 X1,X2,F1,F2,FM,XM,EL,EU
      CCB=.01
      EU=.001D0
      EL=.0001D0
102   XM=(X1+X2)/2.D0
      CALL FUN(XM,FM,EL,EU,NC,NN)
      IF(F1*FM.LE.0.D0) GO TO 100
      X1=XM
      F1=FM
      GO TO 101
100   X2=XM
      F2=FM
101   RE=DABS(X1-X2)/X1
      IF(RE.GT.CCB) GO TO 102
      RETURN
      END
```

A.5 Subroutine FUNEV(K,X,Y,F)

Description. This subroutine is called by RK7. It describes the system of differential equations for the different orders of the Taylor series.

Parameters. K is the order of the Taylor series term. F is the matrix of the derivative values. Y is the matrix of the fundamental solution. W is the frequency.

```
      SUBROUTINE FUNEV(K,X,Y,F)
C     THIS SUBROUTINE SUPPORTS THE RUNGE KUTTA AND DESCRIBES THE
C     DIFFERENTIAL EQN
C
C     THIS SUB WILL NEED CHANGING IF SEI IS TO HAVE DERIVATIVES
C     OR IF THE TORSIONAL MODES ARE BEING DETERMINED
C**** K=ORDER OF THE TAYLOR SERIS TERMS
C**** F=DERIVATIVES VALUES
C**** Y=FUNDAMENTAL SOLUTION
C**** W=FREQUENCY
      REAL*8 F,Y,DRM,W,SEI,X
      DIMENSION F(8,13),Y(8)
      COMMON/EGNVL/W
      F(1,K)=Y(2)
      F(2,K)=Y(3)
      F(3,K)=Y(4)
      F(4,K)=DRM(X)*W**2*Y(1)/SEI(X)
      F(5,K)=Y(6)
      F(6,K)=Y(7)
      F(7,K)=Y(8)
      F(8,K)=DRM(X)*W**2*Y(5)/SEI(X)
      RETURN
      END
```


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A.6 Function SEI(X)

Description. This function determines the bending stiffness of the wing at any point of its span. SEI can be changed to fit any kind of wing.

Parameters. X is the distance from the semi-span.

```
      FUNCTION SEI(X)
C*** X=DISTANCE FROM SEMI SPAN
      IMPLICIT REAL*8 (A-H,O-Z)
      SEI=900000000.D0
       IF(X.GE.11.D0) SEI=90000000.D0
      RETURN
      END
```

A.7 Subroutine SECANT(X1,F1,X2,F2,NC,NN)

Description. This subroutine improves on the approximation given by BISECTION by decreasing the error bound on two successive values of the characteristic determinant.

Parameters. X1 and X2 are two consecutive values in the frequency domain; F1 and F2 are the corresponding values of the characteristic determinant. EL and EU are the error bounds on the values of the calculated solution of RK7.

```
      SUBROUTINE SECANT(X1,F1,X2,F2,NC,NN)
      REAL*8 X1,F1,X2,F2,XM1,FP,DX,XM,FM,CCS,EU,EL
      EL=.000000001D0
      EU=.00000001D0
      CCS=.0001D0
      XM1=X1
      CALL FUN(X1,F1,EL,EU,NC,NN)
      CALL FUN(X2,F2,EL,EU,NC,NN)
103   FP=(F2-F1)/(X2-X1)
      DX=-F1/FP
      XM=X1+DX
      CALL FUN(XM,FM,EL,EU,NC,NN)
      IF((DABS(XM-XM1)/XM).LT.CCS) GO TO 101
      IF(DABS(FM).LT.CCS) GO TO 101
      IF(FM*F1.LE.0.D0) GO TO 500
      X1=XM
      F1=FM
      XM1=XM
      GO TO 103
500   X2=XM
      F2=FM
      XM1=XM
      GO TO 103
101   X2=XM
      F2=FM
      RETURN
      END
```


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A.8 Subroutine DMODE(YY,NN)

Description. This subroutine calculates the values of the modes at 151 points of the semi-span. These values are normalized such that the value of the mode at the wing tip is unity.

Parameters. YY is the matrix of the values of the mode at NN = 151 points of the semi-span.

```
      SUBROUTINE DMODE(YY,NN)
      REAL*8 Y,X,C1,C2,D
      COMMON/FACT/Y(8,151),X(151)
      DIMENSION YY(151)
      D=Y(1,NN)*Y(7,NN)-Y(5,NN)*Y(3,NN)
      C1=Y(7,NN)/D
      C2=-Y(3,NN)/D
      DO 100 I=1,NN
100   YY(I)=C1*Y(1,I)+C2*Y(5,I)
      RETURN
      END
```

A.9 Function DRM(X)

Description. This function gives the mass distribution along the semi-span as given in Figure 3.

Parameters. X is the distance from the semi-span.

```
      FUNCTION DRM(X)
      IMPLICIT REAL*8 (A-H,O-Z)
      DRM=130.D0
      IF(X.LE.4.D0) DRM=2205.D0
      IF(X.LE.11.D0.AND.X.GE.8.D0) DRM=2600.D0
      RETURN
      END
```

A.10 Function RM(X)

This function is the same as DRM(X).

```
      FUNCTION RM(X)
      RM=130.
      IF(X.LE.4.) RM=2205.
      IF(X.LE.11..AND.X.GE.8.) RM=2600.
      RETURN
      END
```


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A.11 Function SIMP(Y,H,N)

Description. This function is called by COF to integrate the functions to calculate the generalized masses and the aerodynamic cross terms of the modes.

Parameters. Y is the mode matrix, H is the distance between two consecutive nodes, and N is the number of nodes.

```
      FUNCTION SIMP(Y,H,N)
      DIMENSION Y(N)
      T1=0.
      J1=N-2
      T2=0.
      DO 100 I=3,J1,2
100   T1=T1+Y(I)
      DO 200 I=2,N,2
200   T2=Y(I)+T2
      SIMP=H*(Y(1)+Y(N)+2.*T1+4.*T2)/6.
      RETURN
      END
```

A.12 Subroutine COF(N,YY,W2,NN,I1)

Description. This subroutine calculates the generalized masses and the aerodynamic cross terms of the modes. The rigid-body translation and rolling and the first four elastic modes are taken into account.

Parameters. RP is the array of the six modes dealt with here. RP(1,I) represents the rigid-body translation and RP(6,I) represents the rigid-body rolling. A and B are matrices for the aerodynamic cross terms. RA is a matrix representing the semi-chord distribution along the semi-span.

```
      SUBROUTINE COF(N,YY,W2,NN,I1)
C     THIS SUBROUTINE SETS UP THE FUNCTIONS FOR INTEGRATION
C***  YY=NEW MODE THEN LATER USED AS SCRATCHED ARRAY
C***  RP=ARRAY OF MODES
C***  W=FREQUENCY
C***  GM=GENERALIZED MASS
C***  A & B=AREODYNAMMIC CROSS TERMS
C***  RA=WING PLAN
C***  N=NUMBER OF MODE BEING WROKED ON
C***  NN=NUMBER OF NODES
C***  I1=NUMBER OF TIMES COF CALLED
      COMMON/DATARA/RA(151)
      COMMON/TRAN1/W(6),GM(6),RP(6,151),A(6,6),B(6,6)
      DIMENSION YY(151),XX(151),RR(151)
C***  READ DATA
      H=33./(NN-1)
      IF(I1.GT.1) GO TO 500
      DO 601 I=1,NN
 601  XX(I)=(I-1)*H
```


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```
      NM=N-1
      W(1)=0.
      GM(1)=40000.0
      DO 600 I=1,NN
  600 RP(1,I)=1.
      A(1,1)=45.038925
      B(1,1)=52.939763
C***  CALCULATE NEW DATA
  500 CONTINUE
      DO 101 I=1,NN
  101 RP(N,I)=YY(I)
      DO 102 I=1,NN
      A1=RM(XX(I))
      B1=RP(N,I)*RP(N,I)
      YY(I)=A1*B1
  102 CONTINUE
      GM(N)=SIMP(YY,H,NN)*2.
      W(N)=W2
      DO 104 J=1,N
        IF(N.EQ.6)GOTO 105
      NR2=N/2.
      NR1=J/2.
      N2=N-2*NR2
      N1=J-2*NR1
      IF(N2.EQ.0.AND.N1.EQ.0) GO TO 400
      IF(J.EQ.1) GO TO 400
      IF(N2.EQ.1.AND.N1.EQ.1) GO TO 400
      A(N,J)=0.0
      B(N,J)=0.0
      GO TO 105
  400 CONTINUE
      DO 103 I=1,NN
      YY(I)=RA(I)*RP(J,I)*RP(N,I)
  103 RR(I)=RA(I)*YY(I)
      A(N,J)=SIMP(RR,H,NN)
      B(N,J)=SIMP(YY,H,NN)
  105 CONTINUE
        IF(N.NE.6)GOTO 107
        A(6,1)=0.0
        A(6,2)=0.0
        A(6,4)=0.0
        IF(J.EQ.1)GOTO 107
        IF(J.EQ.2)GOTO 107
        IF(J.EQ.4)GOTO 107
        DO 108 I=1,NN
        YY(I)=RA(I)*RP(J,I)*RP(N,I)
  108   RR(I)=RA(I)*YY(I)
        A(6,J)=SIMP(RR,H,NN)
        B(6,J)=SIMP(YY,H,NN)
  107   CONTINUE
  104   CONTINUE
        RETURN
      END
```

A.13 Subroutine STRK(NN)

Description. This subroutine sets up all the coefficients needed in RK7 to solve the system of differential equations using Runge-Kutta method of order seven. It also describes the semi-chord distribution along the semi-span. The semi-chord distribution is illustrated in Figure 4.

Parameters. A, B, and C are the coefficient matrices. RA is a matrix representing the semi-chord distribution at 151 points. X is the abscissa matrix of the mode values.

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```
      SUBROUTINE STRK(NN)
C     THIS SUBROUTINE INITIALIZES THE COEFICI
C     FIXES THE NODES AND WING PLAN
C**** NN=NUMBER OF NODES
C**** A,B,C = COEFFICINTS FOR RK7
C**** X=NODE VALUES
C**** RA=WING PLAN ARRAY
      REAL*8 A,B,C,CH,X,Y,DEL
      COMMON/DATARA/RA(151)
      COMMON/RKC/A(13),B(13,12),C(13),CH(13)
      COMMON/FACT/Y(8,151),X(151)
      DEL=33.D0/(NN-1)
      DEL2=33./(NN-1)
      ND=(NN-1)/3.
      RM1=(NN-1)/3.
      DO 100 I=1,NN
      X(I)=(I-1)*DEL
      RA(I)=1.0
      IF(I.GT.ND)RA(I)=1.-.027*(I-RM1)*DEL2
  100 CONTINUE
      A(1)=0.0D0
      A(2)=2.D0/27.D0
      A(3)=1.D0/9.D0
      A(4)=1.D0/6.D0
      A(5)=5.D0/12.D0
      A(6)=1.D0/2.D0
      A(7)=5.D0/6.D0
      A(8)=1.D0/6.D0
      A(9)=2.D0/3.D0
      A(10)=1.D0/3.D0
      A(11)=1.D0
      A(12)=0.D0
      A(13)=1.D0
      B(1,1)=0.0D0
      B(2,1)=2.D0/27.D0
      B(3,1)=1.D0/36.D0
      B(3,2)=1.D0/12.D0
      B(4,1)=1.D0/24.D0
      B(4,2)=0.D0
      B(4,3)=1.D0/8.D0
      B(5,1)=5.D0/12.D0
      B(5,2)=0.0D0
      B(5,3)=-25.D0/16.D0
      B(5,4)=25.D0/16.D0
      B(6,1)=1.D0/20.D0
      B(6,2)=0.D0
      B(6,3)=0.D0
      B(6,4)=1.D0/4.D0
      B(6,5)=1.D0/5.D0
      B(7,1)=-25.D0/108.D0
      B(7,2)=0.D0
      B(7,3)=0.0D0
      B(7,4)=125.D0/108.D0
      B(7,5)=-65.D0/27.D0
      B(7,6)=125.D0/54.D0
      B(8,1)=31.D0/300.D0
      B(8,2)=0.0D0
      B(8,3)=0.D0
      B(8,4)=0.D0
      B(8,5)=61.D0/225.D0
      B(8,6)=-2.D0/9.D0
      B(8,7)=13.D0/900.D0
      B(9,1)=2.D0
      B(9,2)=0.D0
      B(9,3)=0.D0
      B(9,4)=-53.D0/6.D0
      B(9,5)=704.D0/45.D0
      B(9,6)=-107.D0/9.D0
      B(9,7)=67.D0/90.D0
      B(9,8)=3.D0
      B(10,1)=-91.D0/108.D0
      B(10,2)=0.0D0
      B(10,3)=0.D0
      B(10,4)=23.D0/108.D0
      B(10,5)=-976.D0/135.D0
      B(10,6)=311.D0/54.D0
      B(10,7)=-19.D0/60.D0
      B(10,8)=17.D0/6.D0
      B(10,9)=-1.D0/12.D0
      B(11,1)=2383.D0/4100.D0
      B(11,2)=0.D0
      B(11,3)=0.D0
      B(11,4)=-341.D0/164.D0
      B(11,5)=4496.D0/1025.D0
      B(11,6)=-301.D0/82.D0
      B(11,7)=2133.D0/4100.D0
      B(11,8)=45.D0/82.D0
      B(11,9)=45.D0/164.D0
      B(11,10)=18.D0/41.D0
      B(12,1)=3.D0/205.D0
      B(12,2)=0.D0
      B(12,3)=0.D0
      B(12,4)=0.D0
      B(12,5)=0.D0
```


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```
      B(12,6)=-6.D0/41.D0
      B(12,7)=-3.D0/205.D0
      B(12,8)=-3.D0/41.D0
      B(12,9)=3.D0/41.D0
      B(12,10)=6.D0/41.D0
      B(12,11)=0.0D0
      B(13,1)=-1777.D0/4100.D0
      B(13,2)=0.D0
      B(13,3)=0.D0
      B(13,4)=-341.D0/164.D0
      B(13,5)=4496.D0/1025.D0
      B(13,6)=-289.D0/82.D0
      B(13,7)=2193.D0/4100.D0
      B(13,8)=51.D0/82.D0
      B(13,9)=33.D0/164.D0
      B(13,10)=12.D0/41.D0
      B(13,11)=0.D0
      B(13,12)=1.D0
      C(1)=41.D0/840.D0
      C(2)=0.D0
      C(3)=0.D0
      C(4)=0.D0
      C(5)=0.D0
      C(6)=34.D0/105.D0
      C(7)=9.D0/35.D0
      C(8)=9.D0/35.D0
      C(9)=9.D0/280.D0
      C(10)=9.D0/280.D0
      C(11)=41.D0/840.D0
      C(12)=0.D0
      C(13)=0.D0
      CH(1)=0.D0
      CH(2)=0.D0
      CH(3)=0.D0
      CH(4)=0.D0
      CH(5)=0.D0
      CH(6)=34.D0/105.D0
      CH(7)=9.D0/35.D0
      CH(8)=9.D0/35.D0
      CH(9)=9.D0/280.D0
      CH(10)=9.D0/280.D0
      CH(11)=0.D0
      CH(12)=41.D0/840.D0
      CH(13)=41.D0/840.D0
      RETURN
      END
```

A.14 Subroutine RK7(NS,NN,EL,EU)

Description. This subroutine is used to calculate the solution of the differential equation for a given frequency. It uses the Runge-Kutta-Fehlberg method seven and eight order.

Parameters. EL and EU are lower bound and upper bound for the error. NS is the number system of the equations. Y is the solution matrix.

```
      SUBROUTINE RK7(NS,NN,EL,EU)
C*** RUNGE KUTTA FEHLBERG SEVENTH ORDER
C*** EL=ERROR LOWER BOUND
C*** EU=ERROR UPPER BOUND
C*** NS=NUMBER OF SYSTEM OF EQN
C*** Y=SOLUTION
C*** NN=NUMBER OF PTS TO DETERMINE THE SOLUTION
C*** RL=LENGTH OF INTERVAL
      DIMENSION YO(8),F(8,13),YY(8)
      DIMENSION DY4(8),DY5(8),Y1(8)
      COMMON/RKC/A(13),B(13,12),C(13),CH(13)
      COMMON/FACT/Y(8,151),X(151)
      REAL*8 DY4,DY5,XX,YY,TH,A,B,C,F,DD1,DD2,H,Y1,YO,CH,EL,EU,X,Y
      DO 101 I=1,NS
  101 YO(I)=Y(I,1)
```


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```
      NT=NN-1
      L=1
C*** MAIN DO LOOP INCREAMENT TO EACH NODE
      DO 100 I1=1,NT
      NC=0
      H=X(I1+1)-X(I1)
      GO TO 203
  207 L=L-1
      GO TO 203
  206 L=L+1
      NC=1
  203 DO 201 I=1,NS
  201 Y1(I)=YO(I)
      TH=H/L
C*** DO LOOP FOR STEPS BETWEEN NODES
      DO 200 I2=1,L
C*** DETERMINE THE NEEDED FUNCTION EVALURATION
      DO 300 K=1,13
      KM=K-1
      DO 301 J=1,NS
      XX=X(I1)+TH*(I2-1)+A(K)*TH
      YY(J)=Y1(J)
      IF(KM.EQ.0) GO TO 303
      DO 302 I3=1,KM
  302 YY(J)=TH*B(K,I3)*F(J,I3)+YY(J)
  303 CONTINUE
  301 CONTINUE
  300 CALL FUNEV(K,XX,YY,F)
C*** DETERMINE SOLUTION VALUE FOR END OF STEP
      DO 500 I=1,NS
      DY4(I)=0.0
  500 DY5(I)=0.0
      DO 401 I=1,NS
      DO 402 K=1,13
      DY4(I)=TH*C(K)*F(I,K)+DY4(I)
  402 DY5(I)=TH*CH(K)*F(I,K)+DY5(I)
  401 CONTINUE
C*** ERROR AND STEP SIZE CONTROL
      DD1=DABS((DY4(1)-DY5(1))/(DY4(1)+Y1(1)))
      DD2=DABS((DY4(5)-DY5(5))/(DY4(5)+Y1(5)))
      IF(DD1.LT.EU.AND.DD2.LT.EU) GO TO 202
      GO TO 206
  202 IF(DD1.GT.EL.AND.DD2.GT.EL) GO TO 204
      IF(L.EQ.1) GO TO 204
      IF(NC.EQ.1) GO TO 207
      GO TO 207
  204 CONTINUE
      DO 205 I=1,NS
  205 Y1(I)=Y1(I)+DY4(I)
  200 CONTINUE
      DO 102 I=1,NS
      YO(I)=Y1(I)
  102 Y(I,I1+1)=YO(I)
  100 CONTINUE
      RETURN
      END
```

A.15 Subroutine D2

Description. This program solves the forced vibration program. It calculates the amplitudes of the different modes due to sinusoidal gust at different stations of the wing and for different gust frequencies. First, it sets the input data and then it solves the system of algebraic equations for different gust locations.

Parameters. NN is the number of modes at which modes are calculated and N2 is the number of gust locations.

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```
      SUBROUTINE D2
C     THIS PROGRAM DETERMINES THE AMPLITUDES OF THE DIFFERENT--
C     MODES TO A SINUSODAL GUST AT THE DIFFERENT STATIONS ALONG
C     THE WING. SUB...DO TAKES CARE OF INPUT AND OUTPUT PLUS
C     SETS UP THE COEEFICIENTS THAT ARE DRIVING FREQUENCY INDEPENDENT.
C     SUBROUTINE COEF SET UP THE COEFFICIENT MATRIX FOR EACH DRIVING
C     FREQUENCY. WHILE SUB...GAUSS DOES HALF OF THE REDUCTION AND
C     SUB...BACKS FINISHES THE REDUCTION AND DOES BACK SUBSITUTION
C     FOR THE DIFFERENT NON-HOMOGENOUS VECTORS CORRESPONDING TO
C     DIFFERENT GUST LOCATIONS.
C*** NN=NUMBER OF NODES THE MODES ARE DETERMINE ON
C*** N2=NUMBER OF GUST LOCATION
      NN=151
      N2=20
      CALL DO(NN,N2)
      RETURN
      END
```

A.16 Subroutine DO(NN,N2)

Description. This subroutine sets the constants needed later in calculations of the mode amplitudes. Then it normalizes the generalized masses and the aerodynamic cross terms, after which different gust frequencies are considered. For each of these frequencies, the coefficient matrix of the unknowns and the nonhomogeneous vectors for 19 gust stations along the semi-span are calculated by calling subroutine COEF. Subroutine GAUSS is called to perform reduction of the coefficient's matrix of the unknowns. Finally, subroutine BACKS is called to perform reduction on the nonhomogeneous vectors and to perform back substitution.

Parameters. Y is the solution of the system of equations, i.e., the amplitudes of modes. RP is the array of the six modes considered. X is the gust location. A and B were arrays of the aerodynamic cross terms and now become the arrays of the normalized aerodynamic cross terms. W is the array of the natural frequencies of modes. GM was the array of the generalized masses of the modes; these values are normalized and transferred to the array GAMA. OMEG is the reduced natural frequency; these frequencies are reduced with respect to the semi-root chord BR and flight speed of the airplane U = 575 ft/sec. RRK(37) is

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the matrix of reduced frequencies of the gust. RO = 0.0765 is the air density. S = 960 ft^2 is the wing area.

```
      SUBROUTINE DO(NN,N2)
C     THIS SUBROUTINE TAKES CARE OF INPUT AND OUTPUT
C     AND PERFORMS OPERATIONS THAT ARE DRIVING FREQUENCY
C     INDEPENDENT, MEANING GAMA AND OMEG.
C*** Y=SOLUTION, AMPLITUDES OF MODES
C*** RP=MODE ARRAY
C*** X=GUST LOCATION
C*** A,B=AERODYNAMIC CROSS PRODUCTS
C*** W=NATURAL FREQUENCIES OF MODES
C*** GM=GENERALIZED MASS OF MODES
C*** OMEG=REDUCED NATURAL FREQUENCY
C*** GAMA=NON DIMENSIONAL GM
      COMMON/SOL/Y(20,6)
      COMMON/TRAN1/W(6),GM(6),RP(6,151),A(6,6),B(6,6)
      COMMON/TRAN2/RRK(37)
      COMMON/TRAN3/SY(37,19,6)
      COMMON/EDAT/EEE
      COMMON/FAC/PI,BR,S,RO,U
      DIMENSION X(20)
      COMPLEX Y,SY
      COMPLEX CMPLX
      COMMON/DAT/GAMA(6),OMEG(6)
      N=37
      PI=3.14159
      BR=19.0/2.0
      S=960.0
      RO=.0765
      U=575.0
C*** PERFORM ARITHMATIC
      DO 601 I=1,6
      GAMA(I)=GM(I)/(PI*RO*S*BR)
      OMEG(I)=W(I)*BR/U
      JN=I
      DO 602 J=1,JN
      A(I,J)=A(I,J)*BR/S
      B(I,J)=B(I,J)*BR/S
      A(J,I)=A(I,J)
      B(J,I)=B(I,J)
 602  CONTINUE
 601  CONTINUE
      DO 500 I=1,37
      IF(I.LE.10) RK=I/1000.
      IF(I.LE.19.AND.I.GT.10) RK=(I-9)/100.
      IF(I.LE.28.AND.I.GT.19) RK=(I-18)/10.
      IF(I.GT.28) RK=(I-27)
      DEL=33./(N2-1)
      N2M=N2-1
      DO 100 I1=1,N2M
 100  X(I1)=DEL*(I1-.5)
      CALL COEF(RK,X,DEL,N2M,NN)
      CALL GAUSS(6,N2M)
      CALL BACKS(N2M,6)
      RRK(I)=RK
      DO 1 J=1,N2M
      DO 2 J2=1,6
      SY(I,J,J2)=Y(J,J2)
 2    CONTINUE
 1    CONTINUE
 102    CONTINUE
 500  CONTINUE
      RETURN
      END
```

A.17 Subroutine COEF(RK,X,DEL,N2M,NN)

Description. This subroutine calculates the matrix of coefficients of the system of linear algebraic equations. The coefficients of the homogeneous system are calculated only once since they do not change for all the gust locations. The nonhomogeneous vector has to be calculated for 38 gust stations along the entire wing.

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Parameters. C is the coefficient matrix of the homogeneous system. D is the array of the nonhomogeneous vectors. RK is the reduced frequency of the gust. X is the matrix of locations of the gust. DEL is the distance between two gust locations. N2M is the number of gust locations.

```
      SUBROUTINE COEF(RK,X,DEL,N2M,NN)
C    THIS SUBROUTINE SETS UP COEFFICIENT MATRIX AND THE DIFFERENT
C    NON HOMOGENOUS VECTORS.
C** C=COEFFICIENT MATRIX
C** D=ARRAY OF NON HOMOGENOUS VECTOR
C*** C=COEFICIENT ARRAY
C*** RK=REDUCED FREQUENCY
C*** X=LOCATION OF GUST
      COMMON/LS/C(6,6),D(20,6)
      COMMON/DAT/GAMA(6),OMEG(6)
      COMMON/TRAN1/W(6),GM(6),RP(6,151),A(6,6),B(6,6)
      COMMON/FAC/PI,BR,S,RO,U
      DIMENSION X(20),S1(6,6)
      COMPLEX C,CI,D,CC,RKK
      COMPLEX CMPLX
C*** READ DATA
      CI=CMPLX(0.,1.)
      DO 103 I=1,6
      DO 104 J=1,6
      C(I,J)=-RK**2*A(I,J)+2*CI*RK*CC(RK)*B(I,J)
      IF(I.NE.J) GO TO 901
      C(I,J)=C(I,J)+GAMA(I)*(OMEG(I)**2-RK**2)
 901  S1(I,J)=CABS(C(I,J))
 104  CONTINUE
 103  CONTINUE
        GOTO 13
13      CONTINUE
      DO 106 J=1,N2M
      DO 105 I=1,6
      D(J,I)=2*BR/S*RA(X(J))*RPH(I,X(J),NN)*RKK(RK)*DEL
 105  CONTINUE
 106  CONTINUE
      RETURN
      END
```

A.18 Complex Function CC(RK)

Description. This function calculates the Theodorsen function in terms of the Bessel functions RJ0, RJ1, RY0, and RY1.

Parameters. CC(RK) is a function of reduced gust frequency RK.

```
      COMPLEX FUNCTION CC(RK)
C     THIS FUNCTION CALCULATES THE THEODORSEN FUNCTIONS
      COMPLEX CI
      COMPLEX CMPLX
      CI=CMPLX(0.,1.)
      PJ1=RJ1(RK)
      PJ0=RJ0(RK)
      PY1=RY1(RK)
      PY0=RY0(RK)
      F=PJ1*(PJ1+PY0)+PY1*(PY1-PY0)
      G=PY1*PY0+PJ1*PJ0
      H=(PJ1+PY0)**2+(PY1-PJ0)**2
      CC=(F+CI*G)/H
      RETURN
      END
```


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A.19 Function RPH(I,Y,N)

Description. This function calculates an average value for the modes stored in RP at a certain gust location.

Parameters. I is the mode number, Y is the gust location, and N is the number of modes.

```
      FUNCTION RPH(I,Y,N)
      COMMON/TRAN1/W(6),GM(6),RP(6,151),A(6,6),B(6,6)
       DEL=33./(N-1)
      NN=ABS(Y)/DEL+1
      RS=ABS(Y)/DEL+1.-NN
      RPH=RP(I,NN)+RS*(RP(I,NN+1)-RP(I,NN))/DEL
      IF(Y.LT.0.0) GO TO 500
      GO TO 600
500   CONTINUE
      IF(I.EQ.3.OR.I.EQ.5) RPH=-RPH
        IF(I.EQ.6)RPH=-RPH
600   RETURN
      END
```

A.20 Function RA(Y)

Description. This function describes the semi-chord distribution along the semi-span.

Parameters. Y is the distance from mid-span.

```
      FUNCTION RA(Y)
      RA=1-.027*(ABS(Y)-11.)
      IF(ABS(Y).LE.11) RA=1.
      RETURN
      END
```

A.21 Function RY1(X)

Description. This function is the Bessel function of the second kind order one.

Parameters. X is the reduced frequency of the gust.

```
      FUNCTION RY1(X)
      Z=(X/3.)**2
      RY1=((((((.0027873*Z-.0400976)*Z+.3123951)*Z-1.3164827)*Z
     ++2.1682709)*Z+.2212091)*Z-.6366198+.6366198*X*ALOG(X/2.)*RJ1(X)
     +)/X
      RETURN
      END
```


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A.22 Complex Function RKK(RK)

Description. This function is the Kussner function. It is calculated from Theodorsen function CC(RK) and Bessel functions RJ0 and RJ1.

Parameters. RKK(RK) is a function of reduced gust frequency RK.

```
      COMPLEX FUNCTION RKK(RK)
C     FUNCTION DETERMINES THE GUST FORCE FUNCTION
      COMPLEX CI,CC
      COMPLEX CMPLX
      CI=CMPLX(0.,1.)
      PJ1=RJ1(RK)
      RKK=CC(RK)*(RJ0(RK)-CI*PJ1)+CI*PJ1
      RETURN
      END
```

A.23 Function RJ1(X)

Description. This function is the Bessel function of first kind order one.

Parameters. RJ1 is a function of reduced gust frequency RK.

```
      FUNCTION RJ1(X)
      Z=(X/3.)**2
      RJ1=((((((.00001109*Z-.00031761)*Z+.00443319)*Z-.03954289)*Z
     ++.21093573)*Z-.56249985)*Z+.5)*X
      RETURN
      END
```

A.24 Function RJ0(X)

Description. This function is the Bessel function of first kind order zero.

Parameters. RJ0 is a function of reduced gust frequency RK.

```
      FUNCTION RJ0(X)
      RJ0=(((((.00021*Z-.0039444)*Z+.0444479)*Z-.3163866)*Z+1.265620B)*
     +Z-2.2499997)*Z+1.0
      RETURN
      END
```

A.25 Function RY0(X)

Description. This function is the Bessel function of second kind of order zero.

Parameters. RY0 is a function of reduced gust frequency RK.

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```
      FUNCTION RY0(X)
      Z=(X/3.)**2
      RY0=(((((-.00024846*Z+.00427916)*Z-.04261214)*Z+.25300117)*Z
     +-.74350384)*Z+.60936)*Z+.367466914.6366198*RJ0(X)*ALOG(X/2.)
      RETURN
      END
```

A.26 Subroutine GAUSS(N,N2)

Description. This subroutine performs Gaussian elimination on the coefficients matrix of the homogeneous system.

Parameters. N is the number of equations and N2 is the number of

```
      SUBROUTINE GAUSS(N,N2)
C     THIS SUBROUTINE DOES GAUSSIAN ELIMINATION FOR ONLY THE COEFFICIENT
C     MATRIX.   SCALED PARTIAL PIVOTING IS USED.
      COMMON/LS/C(6,6),D(20,6)
      COMMON/PIVOT/IPEN(6)
      DIMENSION S(6)
      COMPLEX C,D
C*** K=PIVOT INDEX
C*** C=COFFICIENT ARRAY
C*** D=INHOMOGENOUS VECTOR
C*** N=NUMBER OF EQN
      DO 103 I=1,N
      IPEN(I)=I
      S(I)=0.
      DO 104 J=1,N
  104 IF(CABS(C(I,J)).GT.S(I)) S(I)=CABS(C(I,J))
  103 CONTINUE
      NM=N-1
      DO 100 KK=1,NM
      IS=KK+1
      IP=IPEN(KK)
      J=KK
      CM=CABS(C(IP,KK))/S(IP)
      DO 105 I=IS,N
      IP=IPEN(I)
      T=CABS(C(IP,KK))/S(IP)
      IF(T.LE.CM) GO TO 105
      CM=T
      J=I
  105 CONTINUE
      IPK=IPEN(J)
      IPEN(J)=IPEN(KK)
      IPEN(KK)=IPK
      DO 101 II=IS,N
       I=IPEN(II)
       K=IPEN(KK)
      C(I,KK)=C(I,KK)/C(K,KK)
      DO 102 J=IS,N
  102 C(I,J)=C(I,J)-C(I,KK)*C(K,J)
  101 CONTINUE
  100 CONTINUE
      RETURN
      END
```

A.27 Subroutine BACKS(N1,N2)

Description. This subroutine performs reduction on all the non-homogeneous vectors corresponding to the gust locations on the semi-span. Then it makes back substitution and calculates the unknowns.

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Parameters. N1 = 19 is the number of the nonhomogeneous vectors each corresponding to a gust location along the semi-span. N2 = 6 is the dimension of the nonhomogeneous vectors.

```
      SUBROUTINE BACKS(N1,N2)
C     DOES REDUCTION ON THE NON HOMOGENOUS VECTOR AND THEN DOES
C     BACK SUBSITUTION.
C*** N1=NUMBER OF NON HOMOGENOUS VECTOR
C*** N2=DIMENSION OF NON HOMOGENOUS VECTOR
      COMMON/PIVOT/IPEN(6)
      COMMON/LS/C(6,6),D(20,6)
      COMMON/SOL/Y(20,6)
      COMPLEX C,D,Y
C*** K1=SOLUTION INDEX
C*** REDUCTION ON NON HOMOGENOUS VECTOR
      DO 100 K1=1,N1
      IP=IPEN(1)
      Y(K1,1)=D(K1,IP)
      DO 101 KK=2,N2
      K=IPEN(KK)
      T=0.0
      JN=KK-1
      DO 102 J=1,JN
  102 T=C(K,J)*Y(K1,J)+T
  101 Y(K1,KK)=D(K1,K)-T
      Y(K1,N2)=Y(K1,N2)/C(K,N2)
C*** BACK SUBSITUTION
      JJ=N2
      DO 103 K=2,N2
      JS=JJ
      JJ=JJ-1
      KK=IPEN(JJ)
      T=0.0
      DO 104 J=JS,N2
  104 T=C(KK,J)*Y(K1,J)+T
  103 Y(K1,JJ)=(Y(K1,JJ)-T)/C(KK,JJ)
  100 CONTINUE
      RETURN
      END
```

A.28 Subroutine D3

Description. This program calculates the power spectrum of the wing tip velocity. After assigning a turbulence scale it calls subroutine SPEC to calculate the output spectrum corresponding to this scale.

Parameters. N2 is the number of gust locations and N is the number of gust frequencies. TL is the turbulence length scale.

```
      SUBROUTINE D3
C     THIS PROGRAM PERFORMES THE ARTHIMETIC TO DETERMINE WING TIP
C     VELOCITY POWER SPECTRUM.   SUB...SPEC PERFORMS THE CALCULATION
C     AND FUN...TSPEC EVALUATES ATMOSPHERIC TURBULENCE SPECTRUM.
C*** N2=NUMBER OF GUST STATIONS
C*** N=NUMBER FOR DRIVING FREQUENCIES
C*** TL=TURBULENCE LENGTH SCALE
      COMMON/TRAN2/RRR(37)
      COMMON/TRAN3/SY(37,19,6)
      COMMON/EDAT/EEE
      COMPLEX SY
      CALL COEF1
      CALL COEF2
      RR=19./2.
      N=37
      N22=20
      N22=N22-1
```


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```
      N2=N22*2
        TL=2112.
        DO 100 I=1,N
      RK=RRK(I)
      RNU=RK*TL/BR
      SS=0.0
      U=575.
      TS=TSPEC(SS,RNU,U,TL)
        IF(TL.EQ.132.)GOTO 1000
        IF(TL.EQ.660.)GOTO 1001
        IF(TL.EQ.2112.)GOTO 1000
1000    WRITE(30,2020)RK,TS
        GOTO 3060
1001    WRITE(31,2020)RK,TS
        GOTO 3060
1002    WRITE(40,2020)RK,TS
3060    CONTINUE
2020    FORMAT(2E13.6)
      CALL SPEC(RK,RR,TL,N2,I,RP)
 100  CONTINUE
 101  CONTINUE
      RETURN
      END
```

A.29 Subroutine SPEC(RK,RR,TL,N2,IC,RP)

Description. This subroutine determines the output spectrum. The cross spectrum between the output and the input is also calculated.

Parameters. RR is the output spectrum; RK is the reduced natural frequency, RK = W×BR/V; Z is the response to the gust at one station; SS is the separation distance nondimensionalized by the turbulence scale; RNU is a reduced gust frequency, RNU = RK×TL/BR where RK is the gust frequency.

```
      SUBROUTINE SPEC(RK,RR,TL,N2,IC,RP)
C     THIS SUBROUTINE DETERMINES THE SPECTRUM OF THE WING
C     TIP VELOCITY.
C*** RR=TOTAL AIRPLANE RESPONSE
C*** RK=REDUCED FREQUENCY WB/U
C*** Z=RESPONSE TO GUST AT ONE STATION
C*** N2=NUMBER OF GUST STATIONS
      COMMON/TRAN2/RRK(37)
      COMMON/TRAN3/SY(37,19,6)
      DIMENSION Y(20,6),Z(40)
      COMPLEX Y,CI,Z,T,T2,SY,H,Z1,Z2,H1,H2,Z3,Z4
C*** READ DATA
      CI=CMPLX(0.,1.)
      BR=19./2.
      N22=N2/2
      DEL=66./N2
      U=575.
      DO 100 J=1,N22
      DO 2 J2=1,6
      Y(J,J2)=SY(IC,J,J2)
 2    CONTINUE
 100  CONTINUE
C*** DETERMINE PLANES RESPONSE
      DO 101 J=1,N22
      T=0.0
      T2=0.0
      DO 102 I=2,5
      T2=Y(J,I)+T2
      IF(I.EQ.3.OR.I.EQ.5) T2=T2-2.*Y(J,I)
 102  T=Y(J,I)+T
      Z(J)=T2*RK*CI
 101  Z(J+N22)=CI*RK*T
C*** DETERMINE PLANES TOTAL RESONSE
      TT=0.0
      DO 300 I=1,N2
      IS=I-1
```


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```
      T=0.0
      JN=N2-IS
      DO 301 J=1,JN
 301  T=2*REAL(Z(J)*CONJG(Z(J+IS)))+T
      SS=IS*DEL/TL
      RNU=RK*TL/BR
      IF(IS.EQ.0)T=T/2.
 300  TT=TSPEC(SS,RNU,U,TL)*T+TT
      RR=TT
        IF(TL.EQ.132.)GOTO 7
        IF(TL.EQ.660.)GOTO 8
        IF(TL.EQ.2112.)GOTO 7
7       WRITE(31,17644)RK,RR
        GOTO 3040
8       WRITE(20,17644)RK,RR
        GOTO 3040
9       WRITE(41,17644)RK,RR
3040    CONTINUE
C*** DETERMINE PHASE OF PLANES RESPONSE
        T=0.0
      DO 600 J=1,N2
      IS=N2-J
      SS=IS*DEL/TL
      RNU=RK*TL/BR
      T=TSPEC(SS,RNU,U,TL)*2*REAL(Z(J))+T
600     CONTINUE
        RP=T
        IF(TL.EQ.132.)GOTO 17
        IF(TL.EQ.660.)GOTO 18
        IF(TL.EQ.2112.)GOTO 17
17644   FORMAT(2E13.6)
17      WRITE(32,17644)RK,RP
        GOTO 3050
18      WRITE(7,17644)RK,T
        GOTO 3050
19      WRITE(42,17644)RK,T
3050    CONTINUE
      RETURN
      END
```

A.30 Function TSPEC(SS,RNU,U,TL)

Description. This function calculates power spectrum and cross-power spectrum from von Karman spectrum functions. This function is called in SPEC in order to calculate the output and the cross spectrum between the input and the output.

Parameters. SS is a separation distance divided by turbulence scale. RNU = W×TL/BR is the reduced frequency of turbulence. TL is the turbulence length scale.

```
      FUNCTION TSPEC(SS,RNU,U,TL)
C     THIS FUNCTION DETERMINES TURBULENCE CROSS AND POWER
C     SPECTRUM FROM THE VON KARMAN SPECTRUM FUNCTION.
C*** SS=SEPERATION DIVIDED BY TL(TURBLUNCE LENGTH SCALE)
C*** RNU= W*TL/U THE REDUCED FRQUENCY OF TURBULENCE
C*** U=FLIGHT SPEED OR MEAN WIND SPEED
C*** TL=TURBULENCE LENGTH SCALE
      IF(SS.EQ.0.0) GO TO 500
C     CROSS SPECTRUM
      Z=SS*SQRT(1.+(1.339*RNU)**2)/1.339
      TSPEC=TL*.10853/U*(4.78112*SS**(5./3.)/Z**(5./6.)*BSL1(Z)
     +-SS**(11./3.)/Z**(11./6.)*BSL2(Z))
      RETURN
 500  CONTINUE
C     POWER SPECTRUM
      Z=(1.339*RNU)**2
      TSPEC=TL*(1+(8./3.)*Z)/(1+Z)**(11./6.)/3.14159/U
      RETURN
      END
```


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A.31 Subroutine COEF1

Description. This subroutine is needed in order to calculate the coefficients of the polynomial that approximates the modified Bessel function of the second kind of order 5/6.

```
      SUBROUTINE COEF1
C     THIS SUBROUTINE SETS UP THE COEFFIENETS FOR THE POLYNOMIAL
C     APPROXIMATION FOR THE MODIFRIED BESSEL FUNCTION OF THE
C     SECOND KIND 5/6 ORDER.
      COMMON/K13/A(10),B(10),A2(10)
      F=5./6.
      A(1)=1.0/.9405612296
      DO 100 I=1,9
 100  A(I+1)=A(I)/I/(F+I)
      F=1.0-F
      B(1)=1.0/5.56756615
      DO 101 I=1,9
 101  B(I+1)=B(I)/I/(F+I-1.0)
      S=4.*(5./6.)**2
      A2(1)=1.
      DO 200 I=1,9
 200  A2(I+1)=A2(I)*(S-(2*I-1)**2)/8./I
      RETURN
      END
```

A.32 Function BSL1(Z)

Description. This function calculates the modified Bessel function of the second kind of order 5/6.

Parameters. Z is a function of the reduced frequency of the turbulence, $z = (SS/1.339)\sqrt{1 + (1.339RNU)^2}$.

```
      FUNCTION BSL1(Z)
C     THIS FUNCTION EVALUATES THE MODIFRIED BESSEL FUNCTION
C     OF THE SECOND KIND 5/6 ORDER
      COMMON/K13/A(10),B(10),A2(10)
      IF(Z.LE.2) GO TO 100
      Y=1./Z
      BSL1=SQRT(1.5707*Y)*EXP(-Z)*POLY(A2,10,Y)
      RETURN
 100  Y=(Z/2.0)**2.0
      RIP=(Z/2.0)**(5./6.)*POLY(A,10,Y)
      RIN=POLY(B,10,Y)/((Z/2.0)**(5./6.))
      BSL1=(3.141/2/SIN(5.0*3.141/6.0))*(RIN-RIP)
      RETURN
      END
```

A.33 Function POLY(A,N,Z)

Description. This function performs polynomial evaluations. These polynomials are used in BSL1 or BSL2. The coefficients of these polynomials are calculated in COEF1 and COEF2.

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Parameters. A is the matrix of coefficients of the polynomial. N is the number of terms in the polynomial. Y is either 1/2 or $(Z/2)^2$ depending on whether Z is less than or greater than 2.

```
      FUNCTION POLY(A,N,Z)
C     THIS FUNCTION DOES THE POLYNOMIAL EVALUATIONS
      DIMENSION A(N)
      T=A(N)*Z
      NN=N-2
      DO 100 I=1,NN
  100 T=(T+A(N-I))*Z
      POLY=T+A(1)
      RETURN
      END
```

A.34 Function BSL2(Z)

Description. This function evaluates the modified Bessel function of the second kind of order 11/6.

Parameters. Z is a function of the reduced frequency of the turbulence, $Z = (SS/1.339)\sqrt{1 + (1.339RNU)^2}$.

```
      FUNCTION BSL2(Z)
C     THIS FUNCTION EVALUATES THE MODIFIED BESSEL FUNCTION OF THE
C     SECOND KIND 11/6 ORDER.
      COMMON/K23/E(10),G(10),E2(10)
      IF(Z.LE.2) GO TO 100
      Y=1./Z
      BSL2=SQRT(1.5707*Y)*EXP(-Z)*POLY(E2,10,Y)
      RETURN
  100 Y=(Z/2.0)**2.0
      RIP=(Z/2.0)**(11./6.)*POLY(E,10,Y)
      RIN=POLY(G,10,Y)/((Z/2.0)**(11./6.))
      BSL2=(3.141/2/SIN(11.0*3.141/6.0))*(RIN-RIP)
      RETURN
      END
```

A.35 Subroutine COEF2

Description. This subroutine calculates the coefficients for the polynomial approximation of the modified Bessel function of the second kind of order 11/6.

```
      SUBROUTINE COEF2
C     THIS SUBROUTINE SETS UP THE COEFFICIENTS FOR THE POLYNOMIAL
C     APPROXIMATIONS OF THE MODIFIED BESSEL FUNCTION OF THE
C     SECOND KIND 11/6 ORDER.
      COMMON/K23/E(10),G(10),E2(10)
      F=11./6.
C*****  ONE OVER THE GAMMA VALUE OF 1+ORDER  *******
      E(1)=1.0/1.724362254
      DO 100 I=1,9
  100 E(I+1)=E(I)/I/(F+I)
C*****  ONE MINUS THE ORDER OF THE MODFIED BESSEL **
      F=1.0-F
```


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```
      G(1)=1.0/(-6.68107938)
      DO 101 I=1,9
101   G(I+1)=G(I)/I/(F+I-1.0)
      S=4*(11./6.)**2
      E2(1)=1.
      DO 200 I=1,9
200   E2(I+1)=E2(I)*(S-(2*I-1)**2)/8./I
      RETURN
      END
$
```


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1. REPORT NO.	2. GOVERNMENT ACCESSION NO.	3. RECIPIENT'S CATALOG NO.
NASA CR-3571		

4. TITLE AND SUBTITLE	5. REPORT DATE
Analysis of Vibration Induced Error in Turbulence Velocity Measurements from an Aircraft Wing Tip Boom	June 1982
	6. PERFORMING ORGANIZATION CODE
7. AUTHOR(S)	8. PERFORMING ORGANIZATION REPORT #
Safwan H. Akkari and Walter Frost	
9. PERFORMING ORGANIZATION NAME AND ADDRESS	10. WORK UNIT NO.
University of Tennessee Space Institute Tullahoma, Tennessee 37388	M-383
	11. CONTRACT OR GRANT NO.
	NAS8-34627
	13. TYPE OF REPORT & PERIOD COVERED
12. SPONSORING AGENCY NAME AND ADDRESS	
National Aeronautics and Space Administration Washington, D.C. 20546	14. SPONSORING AGENCY CODE

15. SUPPLEMENTARY NOTES
Marshall Technical Monitor: Dennis W. Camp
Final Report
Extension No. 1

16. ABSTRACT

The effect of rolling motion of a wing on the magnitude of error induced due to the wing vibration when measuring atmospheric turbulence with a wind probe mounted on the wing tip is investigated. The wing considered has characteristics similar to that of a B-57 Cambera aircraft, and Von Karman's cross spectrum function is used to estimate the cross-correlation of atmospheric turbulence. Although the error calculated is found to be less than that calculated when only elastic bendings and vertical motions of the wing are considered, it is still relatively large in the frequency's range close to the natural frequencies of the wing. Therefore, it is concluded that accelerometers mounted on the wing tip are needed to correct for this error, or the atmospheric velocity data must be appropriately filtered.

17. KEY WORDS	18. DISTRIBUTION STATEMENT
Aviation Meteorology Aviation Safety Aircraft Motions Aircraft Measurements Turbulence Spectra	Unclassified - Unlimited Subject Category 47

19. SECURITY CLASSIF. (of this report)	20. SECURITY CLASSIF. (of this page)	21. NO. OF PAGES	22. PRICE
Unclassified	Unclassified	71	A04

For sale by National Technical Information Service, Springfield, Virginia 22161

NASA-Langley, 1982